Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

GENERAL

MECHANICS OF SOLIDS

MECHANICS OF FLUIDS

HEAT

MISCELLANEOUS

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v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	"	ɩ	'	è	yu	ya

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Applied Mechanics Reviews

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December 1950 Vol. 3, No. 12

Communications

Correction to Rev. 1613 (August 1950):

Title should read: **Mayo D. Hersey, Basic principles of lubrication; part 1: Fundamentals, part 2: Engineering applications,** in **J. B. Rather, W. C. Hadley, and Mayo D. Hersey, Diesel lubricating oils and basic principles of lubrication,** Trans. ASME 71 (Feb. 1949), pp. 19–42.

Theoretical and Experimental Methods

2563. **Computation Laboratory, National Bur. of Standards, Table of the Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments,** New York: Columbia Univ. Press, 1950, xi + 427 pp. 19.6 × 26.5 cm., \$7.50.

Principal part of this important volume is a ten-decimal place table of the Bessel functions of the second kind $Y_0(z)$ and $Y(z)$ for complex arguments $z = \rho e^{i\varphi}$, where $\rho = 0.00\,(0.01)\,10$, $\varphi = 0°(5°)\,90°$. These functions have a logarithmic singularity in neighborhood of $z = 0$. To facilitate interpolation in this region, volume also contains a ten-decimal place table of the auxiliary functions

$$Y_0(z) - \frac{2}{\pi} J_0(z) \ln\rho, \text{ and } Y_1(z) - \frac{2}{\pi} J_1(z) \ln\rho + \frac{2}{\pi\rho} e^{-i\varphi}$$

where $\rho = 0.00\,(0.01)\,0.5$, $\varphi = 0°\,(5°)\,90°$. Also, book contains a table of complex zeros of Bessel functions and related values. (Volume is a companion one to "Tables of the Bessel functions of the first kind $J_0(z)$ and $J_1(z)$," prepared and published by the same group.)

Y. L. Luke, USA

2564. **Charles Darwin, Some conformal transformations involving elliptic functions,** Phil. Mag. (7) 41, 1–11 (1950).

Author gives (mostly without proof) explicit formulas for analytic functions yielding conformal maps of a rectangle onto a variety of domains which may be of interest in applications. All formulas involve Jacobian elliptic functions.

Courtesy of Mathematical Reviews Z. Nehari, USA

2565. **E. W. Parkes, Linear simultaneous equations. Some practical aspects of their solution in respect to the time involved with a series and the relative accuracy of the results,** Aircraft Engrg. 22, 48, 56 (1950).

Author investigates time T_n required to solve n linear simultaneous equations in n unknowns and concludes that, for sufficiently large values of n, $T_n \backsimeq yn^3$, where y is a constant of the order of 0.001 hr for electrical calculating machines. He also examines attainable accuracy and concludes that there is a loss of $0.3n$ significant figures between initial coefficients and values that are finally found for the unknowns.

Courtesy Mathematical Reviews W. E. Milne, USA

2566. **Wilhelm Patz, On the equation $x^2 - Dy^2 = \pm C_p$ $(2^{31} - 1)$, where C is as small as possible** (in German), S.-B. Math.-Nat. Kl. Bayer, Akad. Wiss. 1948, 21–30 (1949).

Author illustrates by three examples a continued fraction method of solving $x^2 - Dy^2 = cp$. Examples considered are $x^2 + 3y^2 = p$, $x^2 + 5y^2 = 2p$, and $x^2 - 13y^2 = -p$, where $p = 2^{31} - 1$. Fermat's theorem is used to solve $z^2 = D \pmod p$ and then the convergents of the continued fraction for $(D^{1/2} - z)/p$ give solution (x, y).

Courtesy Mathematical Reviews D. H. Lehmer

2567. **J. Heinhold, Mechanical conformal mapping of $w = z^a$ (a rational)** (in German), Arch. Tech. Messen no. 163, J 081-15 (Aug. 1949).

2568. **G. D. McCann and C. H. Wilts, Application of electric-analog computers to heat-transfer and fluid-flow problems,** J. appl. Mech. 16, 247–258 (Sept. 1949).

Author gives a brief description of the analog computer of network type used by California Institute of Technology, together with a discussion of methods for solving partial differential equations of physical systems. Illustrative examples are taken from potential fields, transient heat-flow phenomena, and fluid flow.

In computers of this type it is conventional to replace partial differential by finite difference equations. Simulated system is divided into a finite number of lumps and a circuit concocted which simulates each lump subject to finite difference approximations. Furthermore, only time can be used as a continuous independent variable in such computers, which can be a serious limitation at times. Two basic methods are possible; in first, time is used as a continuous independent variable, and space coordinates simulated by finite differences. The computer in this case gives transient solutions for each lump of system being simulated. In second case, time is also represented as a finite difference, and solution at any given instant found from steady-state response of an approximate circuit.

In the case of nonlinear equations, it is often necessary to arrive at a solution by successive readjustments of the magnitudes of components within computer. In any case, an engineering accuracy of about 5 to 10 per cent can be expected in solution.

Horace M. Trent, USA

2569. **Bernard Mazelsky and Franklin W. Diederich, Two matrix methods for calculating forcing functions from known responses,** Nat. adv. Comm. Aero. tech. Note 1965, 36 pp. (Oct. 1949).

Authors first develop two matrix methods for calculating responses of linear systems to arbitrary forcing functions. First method shows how to construct a pair of matrixes for obtaining responses by approximating integrand of Duhamel integral by parabolic segments. Second method, similar to first, obtains the result by approximating the forcing function with straight line segments or parabolic arc segments. Both of these methods essentially treat the ordinates of function to be operated on as components of a vector. Elements of matrixes which give response by premultiplication of the vector are made up of integrating and differentiating multipliers.

The inverse problem, namely, calculation of forcing function or indicial response from known responses, is solved by inverting

(where possible) the matrixes which solve direct problem. The matrixes involved are fairly simple ones which are either of triangular form, or can be reduced to such form by partitioning. It is shown that first method leads to matrixes which can be inverted only when initial values of indicial response is zero, while matrixes of second method always can be inverted easily.

Authors illustrate compactness and accuracy of these matrix methods by comparison of numerical solutions for some explicit forcing functions with known exact solutions. Both methods are found to be convenient numerically; however, first method, when it can be used for inverse problem, is less time-consuming than second method. K. E. Bisshopp, USA

2570. **D. Yu. Panov, On a generalization of the Bairstow formula** (in Russian), Prikl. Mat. Mekh. 13, no. 3, 331–332 (1949).

Author gives a method for an approximate factorization of polynomials of even degree in the form

$$\lambda^{2n} + a_1\lambda^{2n-1} + a_2\lambda^{2n-2} + \ldots + a_{2n} \approx \prod_{k=1}^{n} (\lambda^2 + p_k\lambda + q_k)$$

The p_k and q_k are rational functions of the a_i but the explicit expression is not given. When $n = 2$, author's method gives Bairstow's formula

$$\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E \approx (\lambda^2 + B\lambda + C)(\lambda^2 + (CD - EB)\lambda/C^2 + E/C).$$

Courtesy of Mathematical Reviews A. W. Goodman, USA

2571. **Aldo Ghizzetti, Flow in a not homogeneous and anisotropic medium,** Ann. Soc. Polo. Math. 22, 195–200 (1950).

Paper concerns uniqueness of solution of general linear elliptic partial differential equation of second order in two variables and with constant coefficients, when the boundary conditions to be satisfied are that normal derivative of solution vanishes on two infinite edges of a semi-infinite strip, the function itself assumes prescribed values upon finite edge, and function vanishes in a suitable fashion at ∞. Author concludes that there is, at most, one such solution of equation if, and only if, certain inequalities are satisfied by five constants occurring in problem. The only relation between title and contents of paper consists in statement that treated problem was proposed to author by a person working in the field described by title (See Rev. 2693).

C. A. Truesdell, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2569, 2809)

2572. **F. Ursell, On the rolling motion of cylinders in the surface of a fluid,** Quart. J. Mech. appl. Math. 2, 335–353 (Sept. 1949).

Methods introduced in author's paper, "On the heaving motion of a circular cylinder on the surface of a fluid" [title source, 2, part 2 (June 1949), Rev 3, 1727] are applied to rolling motion of cylinders of arbitrary symmetrical section in the surface of a fluid. Emphasis is placed generally on slow rolling motions. Problem is formulated as a potential flow problem by assuming that viscosity and surface tension may be neglected. Boundary conditions on free surface require that pressure be constant, and on the surface of cylinder require that fluid velocity be same as normal velocity of cylinder at every point. An x, y coordinate system is selected such that origin is at mean position of center of cylinder, and y increases with depth. A conformal transformation is introduced such that x, y plane is transformed to a ϵ, η plane in a way that region occupied by fluid is mapped conformally on region bounded by $-\pi/2 \gtrless \eta \gtrless \pi/2$, and $\epsilon_0 < \epsilon < \infty$, and the given cylinder becomes the curve $\epsilon = \epsilon_0$. The boundary conditions in the new coordinate system are satisfied by an infinite series of harmonic polynomials in which coefficients are to be determined. A method of successive approximations is developed for accomplishing this.

Applications of theory are made to determine roll axis of a cylinder of elliptic section, the apparent mass in slow heaving motion, and motion of a ship in a seaway. Problem of damping in rolling motion is also discussed.

R. L. Bisplinghoff, USA

2573. **Howard J. Curfman, Jr., and Robert A. Gardiner, Method for determining the frequency-response characteristics of an element or system from the system transient output response to a known input function,** Nat. adv. Comm. Aero. tech. Note 1964, 23 pp. (Oct. 1949).

Report applies methods previously developed by others to problem of determining steady-state frequency response of a dynamic element of a system when its transient response to a given input is known. A semigraphical method of making this conversion to frequency response form is illustrated, utilizing a system with known response characteristics. Reasonable agreement with known characteristics is obtained. Illustrated methods are useful in evaluating dynamic response characteristics of aircraft, automatic control devices, and instruments. Restrictions on use of this conversion method are: (1) Input to and output of dynamic element must tend toward fixed values after a finite time. (2) Output of dynamic element must be a linear function of input.

W. O. Breuhaus, USA

2574. **Cataldo Agostinelli, On the applicability of the Jacobi method in analytical mechanics to anolonome systems,** I, II (in Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. 7, no. 1–4, 93–99 (1949).

The equations of a nonholonomic system are written in the following Hamiltonian form:

$$\omega_k = \partial H/\partial p_h, \quad h = 1,2,\ldots, n-m$$

$$\frac{dp_h}{dt} + \frac{\delta H}{\delta \xi_h} + \sum_{j,k=0}^{n-m} \gamma_{j.hk} \frac{\partial H}{\partial p_j} \frac{\partial H}{\partial p_k} = 0$$

in which for convenience $\omega_0 = \partial H/\partial p_0 = 1$, and $q_0 = t$. These equations must be completed by the equations

$$\dot{q}_i = \sum_{h=0}^{n-m} \beta_{rk}\omega_k \qquad r = 1, \ldots, n$$

which give the velocities in terms of the parameters $\omega_0, \ldots, \omega_{n-m}$. The γ's are certain known functions, and $\delta/\delta\xi_h$ stands for the linear differential operator $\sum_{r=0}^{n} \beta_{rk}\partial/\partial q_r$.

A rather complicated condition is given for the applicability of the method of Jacobi in which the first-order partial differential equation

$$\frac{\delta V}{\delta \xi_0} + H\left(q_0, q_1, \ldots, q_n; \frac{\delta V}{\delta \xi_1} \cdots \frac{\delta V}{\delta \xi_{n-m}}\right) = 0$$

plays the role of the usual Hamilton-Jacobi partial differential equation. D. C. Lewis, USA

2575. **Renato Nardini, On a dissipative system with n degrees of freedom** (in Italian), Atti Accad. Naz. Lincei, Rend. Cl. Sci. Fis. Mat. Nat. 7, no. 5, 224–227 (1949).

Purpose of paper is to find conditions sufficient for stability of linear dissipative system in which potential energy depends explicitly on time. Method involves setting up and differentiating a certain positive definite quadratic form in coordinates and velocities. D. C. Lewis, USA

Gyroscopics, Governors, Servos

(See also Rev. 2573)

2576. C. Mack, **The calculation of the optimum parameters for a following system,** Phil. Mag. 40, no. 308, 922–928 (Sept. 1949).

Author gives a method for calculating mean square error for feedback "following" systems and suggests varying parameters until a minimum is found. These parameters are then adjusted for "optimum" values by use of another criterion which weights errors at later time more heavily than errors at earlier time. Reviewer believes method to be useful, but that such criteria for optimization should be looked at critically to determine suitability for each particular application.

H. Harris, Jr., USA

Vibrations, Balancing

(See also Revs. 2594, 2769)

2577. André Charrueau, **Small vibratory motions of an elastic body with propagation of discontinuities of the first order** (in French), C. R. Acad. Sci. Paris 230, no. 4, 361–362 (Jan. 1950).

Author indicates that problem of motion of a vertical bar when suddenly released under action of gravity can be considered as a case of small vibrations of an elastic body with propagation of discontinuities. A general case of surfaces of discontinuity is treated.

D. L. Holl, USA

2578. John E. Brock, **Some non-linear systems permitting simple harmonic motion,** J. appl. Phys. 21, 238–243 (1950).

Two nonlinear differential equations are constructed which have solutions expressible in terms of sine terms.

P. Franklin, USA

Courtesy of Mathematical Reviews

2579. I. G. Malkin, **Methods of Lyapunov and Poincaré in the theory of nonlinear oscillations (Metodí Lyapunova i Puankare v teorii nelineínkh kolebaniï)** (in Russian), OGIZ, Moscow-Leningrad, 244 pp. (1949).

Those who have perused the Minorsky report on nonlinear mechanics are well acquainted with the fact that this subject has been treated in the Soviet Union by two distinct schools, one in Moscow and the other in Kiev. The work of the first, centered on the institute of oscillations, is fairly well represented by the material in the "Theory of Oscillations" by Andronow and Chaikin (1937); editorially revised English edition, Princeton University Press (1949)]. In the main the investigations of this school have been based upon the mathematically strictly correct work of Poincaré in "Les Méthodes Nouvelles de la Mécanique Celeste," and of Lyapunov ("Problème général de la stabilité du mouvement," reproduced as Ann. Math. Study no. 17).

The procedure is dominated on the analytical side by the small parameter method with approximations known to be convergent. Parallel investigations took place at Kiev, where Krylov and Bogolyubov developed intensively the "pre-Poincaré" methods of the astronomers Glyden and Lindstedt based on approximations, generally divergent according to Poincaré, but nonetheless of much practical value. This second mode of attack is reasonably well represented in the monograph by Krylov and Bogolyubov: "Introduction to nonlinear mechanics" [Kiev (1937); much abridged edition by reviewer: Ann. Math. Study no. 11.]

I. G. Malkin, the author of the present excellent and well-written monograph, is one of the foremost adherents of the Moscow school. The treatment assumes the fundamental existence theorems and deals exclusively with the periodic solutions of systems

$$\frac{dx_s}{dt} = X_s\,(t, x_1, \ldots, x_r) \qquad [1]$$

where X_s is analytic in the x_i and μ, for the x_i in a certain region and μ small, and is continuous and periodic with period 2π in t (but may fail to contain t). Given a "generating" periodic solution $f_s(t)$ of [1] for $\mu = 0$, the Poincaré theory for the existence of neighboring solutions is discussed and considerably extended. The Lyapunov stability criteria for periodic solutions are thoroughly discussed and applied to the problem. The results of second part of Lyapunov's paper (op.cit.), dealing with periodic solutions, are taken up with particular emphasis on Lyapunov systems (author's terminology), i.e., reducible to the form:

$$\frac{dx}{dt} = -\lambda y + X, \quad \frac{dy}{dt} = x + Y$$

$$\frac{dx_s}{dt} = \Sigma\, b_{si} x_i + X_s \qquad [2]$$

where λ and the b_{si} are constants, and X, Y, X_s do not contain t are analytic at the origin and begin with terms of degree at least two. Furthermore, it is assumed that [2] has an integral $H = x^2 + y^2 + W(x_1, \ldots, x_m) + S(x,y,x_1, \ldots, x_m) = \text{const}$, where W is a quadratic form and S is analytic at the origin and begins with terms of degree at least three. Lyapunov systems possess a family of periodic solutions. Systems very near those of Lyapunov are investigated for their periodic solutions. A goodly number of examples, mainly drawn from differential equations of the second order, are carefully treated throughout the monograph.

S. Lefschetz, USA

2580. E. Zabotinskii, **Auto-oscillating systems with two degrees of freedom in the case of multiple frequencies,** Zhurn. Eksper. Teoret. Fiz. 20, 421–426 (1950).

Author applies standard Poincaré-Lyapunov methods to a pair of quasiharmonic oscillations reducible to

$$\ddot{\xi} + \xi = \mu f(\xi, \dot{\xi}, \eta, \dot{\eta}); \quad \ddot{\eta} + n^2 \eta = \mu g(\xi, \dot{\xi}, \eta, \dot{\eta})$$

where μ is small and f, g are holomorphic in their arguments in the vicinity of a periodic solution of the system corresponding to $\mu = 0$. It is assumed that n is an integer which causes departures from general case. Solutions are discussed of the form $\xi = R \cos t + x$; $\eta = P \cos nt + Q \sin xt + y$, where $x, y \to 0$ with μ and have a period $2\pi + \tau$, where again $\tau \to 0$ with μ. Explicit equations are obtained, stability is discussed, and an application is made to an electric system corresponding to $n = 3$.

S. Lefschetz, USA

2581. H. C. Van De Hulst, **On the attenuation of plane waves by obstacles of arbitrary size and form,** Physica Hague 15, 740–746 (Sept. 1949).

Paper gives a theory of diffraction of plane waves by an arbitrary obstacle with particular attention to what happens behind obstacle in exact direction of incident wave. Author finds that "the extinction cross section of any obstacle is -2λ times the imaginary part of the amplitude function for forward scattered light." As a corollary, "the extinction cross section of a *large* obstacle is twice its geometrical cross section." Author observes that this last result has also been found by other (theoretical) workers. In a recent paper [J. appl. Phys. 20, 1110–1115 (1949)] Léon Brillouin has given a thorough analysis of this problem and shows that the factor 2 above is correct for a spherical obstacle of

radius about 10λ; however, for a sphere of radius larger than about 30λ, the factor has the asymptotic value 1 given by geometrical optics. This has been confirmed by laboratory experiments.

P. LeCorbeiller, USA

2582. N. L. Kaidanovsky, The nature of mechanical autovibrations which take place with dry friction (in Russian), J. tech. Fiz. 19, 985–996 (Sept. 1949).

Paper gives results of an experimental investigation carried out with a view to substantiating theory of Chaikin and author [Jour. tech. Phys. USSR III (1933)] according to which autooscillations appear only in the regions in which characteristic of dry friction exhibits falling branches. Author considers the differential equation of motion of a linear system with one degree of freedom in the form

$$m\ddot{x} + h\dot{x} + kx + F(u - \dot{x}) = 0 \qquad [1]$$

where F is a nonlinear function of relative velocity $u - \dot{x}$, u being constant velocity of an auxiliary system through which dry friction reaction F is introduced. Using approximation $F(u - \dot{x}) = F(u) - F'(u)\dot{x} = F(u) + A\dot{x}$, [1] becomes

$$m\ddot{x} + (h + A)\dot{x} + kx + F(u) = 0 \qquad [2]$$

Starting with this equation as a basis for his experiment, writer concludes that if $A < 0$ and $|A| > h$, [2] has an unstable singularity and, therefore, self-excited oscillations take place; if, however, h is increased so that $|A| < h$, these oscillations disappear. Experimental arrangement consisted of a rotating table on which is mounted a disk made of a metal M and a block of another metal N bearing on M with a certain amount of friction; block N was connected to a recording system; a damper of electromagnetic type was associated with the system N so as to be able to vary the coefficient h. All necessary constants were calculated with a sufficient accuracy to ensure quantitative conclusions. Experimental results corroborate preceding theoretical conclusions.

N. Minorsky, France

2583. G. V. Aronovich, On the shimmy theory of an automobile and aircraft (in Russian), Prikl. Mat. Mekh. 13, no. 5, 477–488 (1949).

By shimmy of an automobile or an airplane with tricycle landing gear is meant an oscillation of the front wheels around the vertical. This oscillation is actually a self-excited one and nonlinear. What matters, primarily, is to determine conditions under which shimmy may arise. Most authors, Rocard excepted, have discussed the problem disregarding influence of the motion of vehicle as a whole. In present paper, the complicated system of differential equations for full motion is set up both for an automobile and a plane. The small oscillations for shimmying are discussed through equations of the first approximation (a linear system).

For an automobile, the following two special cases are examined at length: (a) Vehicle remains strictly horizontal and under rectilinear motion; and (b) wheels are laterally rigid. In treatment of plane, similar simplifying assumptions are introduced. Relevant references, mostly accessible: G. Becker, H. Fromm, H. Maruhn, "Schwingungen in Automobillenkungen," Berlin (1931); I. Rocard, "Les méfaits du roulement, autooscillations et instabilités de route," *La Revue Scientifique* 84, no. 15 (1946); D. Sensaud de Lavaud, Comptes rendus 185, t. I, p. 1636; t. II (1927); Fred. Weick, E.S.A.E. Journ. 38, no. 5 (1936); Kontrowitz, NACA Report 686 (1940); Wylie, J. aero. Sci. (Dec. 1939); M. V. Keldysh, Contributions to the Central Aero-Hydrodynamics Institute (in Russian) no. 564 (1945); A. A. Andronow and C. E. Chaikin, "Theory of oscillations," transl. pub. by Princeton Univ. Press (1948); J. H. Greidanus, Nat. LuchtLab. Amsterdam Rap. V. 1038; M. Julien, Actualités scientifiques et industrielles, Paris, Hermann (1935), no. 279; J. I. Neumark, Avtomatica i telemekharika (in Russian) IX, no. 3 (1948); I. I. Metalitsyn, Doklady (in Russian) 61, no. 3 (1948).

S. Lefschetz, USA

2584. F. J. Meister, Calibration and testing of vibrometers (in German), Arch. tech. Messen 160, V170-3 (March 1949).

After a brief discussion of basic static and dynamic calibration criteria, paper describes construction and performance of a few typical mechanical, electrodynamic, and piezoelectric shake tables for testing frequency response of vibrometers.

G. A. Nothmann, USA

2585. I. G. Malkin, Oscillations of quasilinear systems with a nonanalytic characteristic of nonlinearity (in Russian), Akad. Nauk USSR Prikl. Mat. Mekh. 14, 13–22 (1950).

In a recent monograph, author has investigated the quasiharmonic oscillations of a system $\ddot{x} + k^2x = \mu F(t,x,\dot{x})$ where F is periodic and of period 2π in t with a related Fourier series, and is analytic in $x,\dot{x}$ in a certain region. This investigation is here extended to case where F_x, $F_{\dot{x}}$ satisfy a Lipschitz condition.

S. Lefschetz, USA

2586. Henry E. Fettis, A modification of the Holzer method for computing uncoupled torsion and bending modes, J. aero. Sci. 16, 625–634 (Oct. 1949).

Holzer method is applied to determination of uncoupled torsion and bending modes of a continuous nonuniform beam. Holzer method in its original form was applied to systems of lumped parameters. Paper illustrates a process of numerical integration which permits application of Holzer method to continuous systems.

Extensions of Holzer method to continuous systems have been previously suggested by Wyklestad, Targoff, and others. However, scheme of numerical integration suggested by Fettis makes his scheme better suited to continuous systems. In addition, several features are presented which makes the procedure especially adaptable to routine calculations.

R. L. Bisplinghoff, USA

2587. G. Snowball, Electrical methods of inducing and detecting vibration, Engineering 168, 406–408 (Oct. 14, 1949).

Article describes vibration equipment used for exciting and measuring resonant vibrations in steam-turbine condenser tubes. Exciter is a conventional electrodynamic driver. Amplitude and phase of lateral vibration of tube are measured with a capacitance pickup, which has the important advantage of not loading the vibrating system. Unfortunately, most of text is devoted to presenting known theory of electrodynamic and electrostatic drivers whereas but little space is devoted to describing the more novel capacitance pickup and its circuits.

W. P. Welch, USA

2588. Sumiji Fujii, Time-lag vibration (in Japanese), Rep. Inst. Sci. Tech. Univ. Tokyo 1, 80–83 (1947).

Author studies special cases of $\psi(t) = \Phi\{\psi(t - \tau)\}$, where Φ is an operator, e.g., $\psi(t) = \alpha\psi(t - \tau)$ and $\theta'(t) + \theta(t - \tau) = 0$.

Ed.

Wave Motion, Impact

(See also Rev. 2571)

2589. **Joseph B. Woodson, The dynamic response of a simple elastic system to antisymmetric forcing functions characteristic of airplanes in unsymmetric landing impact,** J. appl. Mech. 16, 310–316 (Sept. 1949).

Work of Biot and Bisplinghoff on the theory of transients in undamped elastic structures is summarized. Considered antisymmetric forcing functions are constructed as follows: It is assumed that the force-time relation for first alighting gear is a half sine pulse, while that for the second is identical to the first but displaced along axis of time by some interval, called delay time.

The dynamic responses are computed for three cases of antisymmetric forcing functions resulting from overlapping half sine pulses. Considered delay times are the period of first pulse, one half the period of pulse, and one third the period of pulse. Maximum values of response (dynamic response factors) are plotted as a function of ratio of period of forcing function to natural period of the simple system. It is stated that these assumed shapes are similar to shapes recorded in model drop tests, but no data are given to support this assertion. It is shown that the three shapes produce roughly the same response factors in the range of values considered.

Of course, the relative values of the response factors for the various modes do not indicate relative values of mode stresses. However, paper is valuable because it indicates that shock-induced stresses in the antisymmetric modes of an airplane should not be ignored a priori.

J. Paul Walsh, USA

2590. **F. Biesel, Study of the damping of swellings due to permeability of the ground** (in French), Int. Assn. for Hyd. Str. Res. no. 1–9, 12 pp. (Sept. 1949).

Two-dimensional wave motion in a canal of depth H lying on a bed which is uniformly porous to a depth H' is considered. By making vertical velocity and pressure continuous at top of porous layer, a velocity potential is found which contains the factor $\exp(-Kax)$ or $\exp(-K'bt)$ according as damping is in space or time, $2\pi/a$ and $2\pi/b$ being wave length and period of waves. In the porous layer, velocity is proportional to pressure gradient, constant of proportionality being $-m/\rho g$. Motion is assumed small and irrotational, and the coefficients K and K' small compared with unity. Obtained values are $K = ma \tanh aH'/b(1 + 2aH/\sinh 2aH)\cosh^2 aH$; $K' = ma \tanh aH'/b \cosh^2 aH$.

Results are extended for variable depths. Though phenomenon is negligible in full scale, formulas give an estimate for model experiments where effect may be appreciable.

R. S. Scorer, England

2591. **F. Biesel, Calculation of wave damping in a viscous liquid of known depth** (in French), Houille blanche 4, 630–634 (Sept.-Oct. 1949).

Effect of viscosity on surface waves in a medium of finite depth can be calculated by familiar methods. For a given wave length, theory leads to a fairly intricate equation for the damping coefficient. In present paper, author criticizes certain approximations introduced by Basset on one hand, and by Hough on the other, for the case of small ν (dynamic viscosity). It is pointed out that Basset omits terms which are not negligible in the sense of the analysis. Criticism of Hough's work amounts to a charge of a certain measure of inconsistency rather than discovery of a definite error. Author then introduces approximate assumptions of his own which for most practical purposes lead to results similar to Hough's. (Hough's paper was published in 1896, not 1877 as stated.)

A. Robinson, USA

Elasticity Theory

(See also Revs. 2614, 2656, 2684)

2592. **R. D. Mindlin, Compliance of elastic bodies in contact,** J. appl. Mech. 16, 259–268 (Sept. 1949).

Author first treats problem of determining tangential compliance of two elastic bodies pressed together and loaded with a small tangential force in contact region. Assuming no slip between the two bodies and that surfaces remain everywhere in contact, author concludes on basis of symmetry conditions that changes in normal component of traction in region of contact may be ignored, which is analogous to neglecting tangential tractions in Hertz theory for bodies of unlike elasticities. Also, the contact region shifts uniformly in direction of tangential force without any change in shape or size. Having the displacements in contact area and components of traction outside this area, potential functions of Boussinesq and Cerruti are utilized to obtain expressions for tangential compliance, including case where the two bodies have unlike elasticities. For a circular contact area, the results may also be obtained using Neuber's solution for a hyperbolic notch subjected to shear loading. For both circular and elliptic contact regions, tangential traction is everywhere parallel to direction of applied force, and becomes infinite at edge of contact region. Effect of slip near the edges for a circular contact region is also considered, and it is shown that if tangential load is small compared to product of normal load and coefficient of friction, error due to neglecting slip is small. Assuming no slip, ratio of tangential to normal compliance ranges between one and two, depending on value of Poisson's ratio and on ratio of major to minor axes. An analysis is also given for torsional compliance in cases where a torsional couple acts across contact region. Charts are also presented to facilitate practical computation of both tangential and torsional compliances.

A. M. Wahl, USA

2593. **A. Signorini, On finite deformations of an elastic solid,** Proc. seventh int. Congr. appl. Mech. 4, 237–247 (1948).

To avoid mathematical complications due to rotations of surface elements on which stresses act, author adopts coordinates of a particle in the deformed medium as independent variables. The strain tensor (containing quadratic terms in derivatives of displacement vector) then determines exactly the initial length of any given line element in strained medium.

Author points out that, when finite deformations are considered, there is no linear tensor relation between stress and strain that is exactly derivable from a positive-definite strain-energy function. He proposes a new positive-definite algebraic function of the three invariants of strain tensor and the two Lamé constants as strain-energy density of an isotropic elastic medium. This function leads to quadratic expressions for stress components in terms of strain components. Furthermore, normal stresses remain independent of shearing strains. For small deformations, stress-strain relationship reduces to that of linear elasticity theory.

New theory is shown to lead to consistent results for simple tension, although the relation between axial strain and lateral strain is not a simple proportion. Also, author obtains exact solution of problem of cylindrical bending of a flat plate with the new stress-strain relation. It is shown that middle surface of plate contracts along axis of cylinder if a tension is not applied.

New theory rigorously satisfies all mathematical requirements. It will gain in mathematical interest and in physical significance if proposed strain-energy function can be shown to be unique in the class of functions that yield stress-strain relations with mentioned properties. Also, a comparison of the new theory with other

theories of finite deformation and with experimental data (e.g., Bridgman's tests) would add to interest and value of work.

H. L. Langhaar, USA

2594. E. R. Lapwood, The disturbance due to a line source in a semi-infinite elastic medium, Phil. Trans. roy. Soc. Lond. Ser. A, no. 841, 242, 38 pp. (July 1949).

Paper analyzes transmission of cylindrical pulses originating beneath surface of a semi-infinite elastic solid. Author gives separate consideration to longitudinal and transverse pulses, obtaining in each case exact solutions expressed by means of three displacement potentials which are repeated contour integrals with elementary but complicated integrands. The remainder of paper consists of approximate evaluation and interpretation of these solutions when both source and observer of disturbance are at a shallow depth (or on surface), and observer is distant from source. Analysis is elaborate and difficult. Conclusions are too lengthy for summary here; in each case, author finds a sequence of six different pulses propagated in different ways. He suggests application of his results to theory of earthquakes.

C. A. Truesdell, USA

2595. J. D. Eshelby, Edge dislocations in anisotropic materials, Phil. Mag. 40, no. 308, 903–192 (Sept. 1949).

Method developed by Green [Proc. Camb. phil. Soc. 41, p. 224 (1945)] for anisotropic plates is extended to case of plane strain in an anisotropic material with the symmetry of any of the crystal classes. Results are applied to discussion of an edge dislocation whose axis is an infinite straight line of arbitrary orientation. Explicit formulas are given for distributions of stress and displacement for a number of cases in which the slip plane and Burger's vector are simply related to symmetry axes of material. Energies of dislocations are calculated and applied to problem of dissociation and combination of dislocations. Nabarro's [Proc. phys. Soc. 52, p. 34 (1940)] calculation of width of a dislocation is extended to anisotropic case. Paper presents a valuable tool for use in application of dislocation theory to actual crystals.

W. T. Read, Jr., USA

Experimental Stress Analysis

(See also Rev. 2666)

2596. Handbook of experimental stress analysis, edited by M. Hetényi, John Wiley & Sons, Inc., New York, 1950, 1077 pp., $15.

Stress analysts have been waiting for this handbook for many years, and they will not be disappointed. Never before has so much of the field been covered so thoroughly. Thirty-one contributors shared in writing 18 chapters and 3 appendixes dealing with the fundamentals of elasticity, dimensional analysis, precision of measurements, and, separately, with each of the several techniques used in experimental analysis. An extensive bibliography, including short explanatory comments, follows each chapter.

Some subjects are discussed so completely that they constitute a treatise in themselves. The 120 pages on analogies give the engineer an excellent guide, and what a photoelastician may need has been developed in concise form in 140 pages. Almost 150 pages are taken by 3 chapters on electrical strain gages of resistance, inductance, and capacitance types. However, reviewer could not find any mention of piezoelectric gages.

The description of mechanical strain gages includes some instruments of historical interest. Nothing is said here, however, about the Fischer method for determining stress concentrations by using strain gages of variable gage length, although this method allows use of mechanical strain gages to the full extent of their possibilities. Acoustical strain gages are included in this chapter. These gages are extensively used in some European countries and, because of the advantages they offer in some applications, they probably deserve more than one page of general description.

Chapter on optical methods gives the necessary basis for an understanding of optical instruments, including photoelastic polariscopes. Several instruments are described. Reader will miss, however, any specific mention of Preuss extensometer with which some of the most precise mechanical analyses were made in Germany.

Brittle coatings and brittle materials are dealt with jointly. Although only a few properties of brittle materials are actually used in the study of brittle coatings, it seems to be a sound approach to try to emphasize what both methods have in common. Reviewer thinks that relative importance given to this chapter is too small. Most recent bibliographic reference mentioned here was published in 1947, although other chapters of the book include 1948 and 1949 references. In reference 8 the comment is not correct, since the tests of flat bars with holes were not run under tension, but under bending.

Chapter on x rays deals not only with stress analysis, but also with crack detection. There is another chapter in this field which, strictly speaking, is outside the stress-analysis field, being dedicated to magnetic and ultrasonic methods of crack detection.

Specific applications to dynamics, residual stresses, and structures are treated in separate chapters, and so are the subjects of testing machines and strain rosettes.

Properties of materials, working stresses, and service fractures are very closely related to stress analysis. Reader will appreciate the chapters dealing with them, although the few lines allotted to plasticity seem too sketchy to be of much help in that important subject. Reviewer thinks that grid methods also deserve more space.

Main shortcoming of the handbook is lack of tables and graphs summarizing results obtained to date. Description of methods is the most important part of a good handbook, but the reader also wants to know what has been done with these methods.

None of the authors seems to be aware of Lehr's *Spannungsverteilung in Konstruktionselementen*. This book, published in 1934, was the first attempt at a stress-analysis handbook, and in spite of several errors it has been, up to now, the most useful source of information.

The contributors are: R. B. Allmutt, Charles S. Barrett, B. C. Carter, C. O. Dohrenwend, Thomas J. Dolan, L. H. Donnell, D. C. Drucker, J. R. Forshaw, J. N. Goodier, M. Hetényi, Oscar J. Horger, B. F. Langer, Charles Lipson, C. W. MacGregor, Joseph Marin, John L. Maulbetsch, W. R. Mehaffey, J. H. Meier, Raymond D. Mindlin, F. Mintz, W. M. Murray, Charles H. Norris, J. Ormondroyd, R. E. Peterson, Mario G. Salvadori, W. T. Savage, J. F. Shannon, C. Richard Soderberg, R. D. Specht, S. P. Timoshenko, and John B. Wilbur.

A. J. Durelli, USA

2597. A. L. Biermasz and H. Hoekstra, The measurement of changes in length with the aid of strain gauges, Philips tech. Rev. 11, 23–31 (July 1949).

A description is presented of several bonded fine-wire strain gages (similar to Baldwin SR-4 types) of Philips manufacture, together with a general discussion of their properties, applications, and associated electrical equipment.

Irwin Vigness, USA

2598. David E. Weiss, Properties and uses of resistance strain gages, Nav. Res. Lab. Rep. no. 5-3276, 1–14 (March 1948).

Theory and construction of variable resistance strain gage, as discussed here, should be of benefit to novice to the field. Pre-

sented information is not new, but this is one of few instances where basic principles and practical applications are presented in a single paper. Circuits for measurement of static and dynamic strains are completely described, as well as technique of mounting and calibrating gages.

H. R. Neifert, USA

2599. **L. D. Anderson, Operating manual for the TMB type 13A strain indicator and associated equipment,** David Taylor Model Basin Rep. no. 608, 22 pp. (July 1949).

Described equipment is an electronic system designed to record in sequence 16 static or slowly varying strains. In operation, a commercial oscillator delivers a voltage to buffer amplifier and bridge sections provided for each channel. The modulated carrier voltages from the latter are then delivered to the strain indicator where detection of null balance, phase discrimination, rectification, and other required functions are performed. Finally, a commercial strip-chart potentiometer is employed for recording successively the strain values. Carrier-type strain indicator employed is one previously developed for dynamic strain measurement (same source, no. 565), and modified for present application. Application for which the equipment was actually designed was not investigation of stresses, but one of measuring pressures by means of capsules containing wire gages.

Sensitivity is such that the 11-in. chart width encompasses 200 micro-inches per inch strain. If, however, larger strains must be recorded, a range extender can be set to balance out a part of expected signal so that final resolution on chart remains the same.

Charles W. Gadd, USA

2600. **Orrin H. Clark and Ernest M. Lyman, Note on some plastic flow effects in steel,** J. appl. Phys. 20, 884–885 (Sept. 1949).

A method of employing interference fringe measurements is presented for study of plastic flow in steel. Method is used to determine depth of depression existing at center of a contact circle between two dissimilar steel pins pressed together under load sufficient to induce plastic flow in the softer one.

Harry Schwartzbart, USA

2601. **B. L. Averbach and B. E. Warren, Interpretation of x-ray patterns of cold-worked metal,** J. appl. Phys. 20, 885–886 (Sept. 1949).

Present experimental techniques make possible determination of intensity distribution across an x-ray powder pattern line. Theoretical expressions for the intensity distribution of these lines are given according to work of Stokes. From these expressions and from intensity measurements for a cold-worked and an annealed sample of α-brass, the RMS residual strains are determined along several lattice directions. Article, which is a letter report, points out that treatment of powder pattern lines in terms of Fourier coefficients offers a powerful method for handling and separating cold-work and particle-size line broadening, together with many other effects.

Irwin Vigness, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Rev. 2610)

2602. **Enrico Castiglia, Equations and analytic properties of influence lines in a continuous beam** (in Italian), G. Gen. civ. 87, 423–441 (Sept. 1949).

Paper gives a derivation of general equations of influence lines for shear, bending moment, bending moment at points of support, and reactions for continuous beams on unyielding supports. Two sets of equations are given, one for the field containing section for which influence lines are desired, and one for all other fields. Various relations between shear and moment influence lines are discussed; these can be used either to shorten the calculations or as a check. Two examples are given, one for a beam with constant moment of inertia and one for a beam of variable section. Detailed tables for various equations needed and auxiliary tables giving necessary numerical values are included. They hold for beams of constant section with no more than five fields.

Bruno A. Boley, USA

2603. **B. L. Abramyan, Torsion of prismatic rods with cruciform cross section** (in Russian), Prikl. Mat. Mekh. 13, no. 5, 551–556 (1949).

Paper contains a solution of Saint Venant's torsion problem for a bar of cruciform cross section. Prandtl's torsion function is obtained in form of an infinite series involving hyperbolic and trigonometric functions. Computation of torsional rigidity, based on Prandtl function, is compared with results given by C. Weber's empirical formulas [*Forschungsheft*, p. 249 (1921)] and found to be in good agreement for thin-flanged sections only.

I. S. Sokolnikoff, USA

2604. **Rolf Lamberg, The calculation of continuous beams** (in German), Bautechnik 26, 205–208 (July 1949).

2605. **M. S. G. Cullimore, The shortening effect—a nonlinear feature of pure torsion,** Engng. Struct. (Spec. suppl. to Research), 153–164 (1949).

C. Weber's analysis [VDI Forschungs arbeiten, no. 249, Berlin (1921)] is extended to explain longitudinal stresses and shortening effect which arise in pure torsion and which are neglected in the classical theory. Results are compared with those obtained from experiments with aluminum alloy I- and Z-sections. It is made out that agreement is satisfactory.

Assumption that such effects may not be treated as nonlinear second-order effects is incorrect. They have been deduced with help of theory of finite deformation [B. R. Seth, Phil. Trans. roy. Soc. Lond. 234, 246–264 (1945)]. It is also not correct to say that center of flexure is not affected by these second-order effects. In all asymmetrical loadings torsion solution forms an important part of flexure solution.

B. R. Seth, USA

2606. **W. Freiberger. The uniform torsion of an incomplete tore,** Austral. J. sci. Res. Ser. A 2, 354–375 (Sept. 1949).

Toroidal coordinates are used to study uniform torsion of an incomplete tore. To compare the stresses with those found in a helical spring of small pitch, an exact solution is obtained for case when resultant of stress system over any terminal cross section is a single force acting along axis of tore.

Numerical results are obtained for case when ratio of radius of central line to that of a circular section is 4. It is found that the engineering formula giving axial lengthening or shortening of a helical spring of thin wire and small pitch can be safely used in all cases likely to occur in practice. The variation of ratio of maximum stress to external force with dimensions of tore confirms approximate results of A. M. Wahl [J. app. Mech. (March 1935)].

B. R. Seth, USA

2607. **A. Cattin, A continuous beam bent and axially loaded by a linearly variable force** (in Italian), G. Gen. civ. 87, 458–465 (Sept. 1949).

A continuous beam carrying a load that varies linearly between any two supports is analyzed for the condition in which load has both axial and flexural components. Analysis, which involves a series of expedient changes of variable, results in a "three-mo-

ment" equation. Latter provides for relative settlement of supports, and reduces to familiar Clapeyron equation in limiting case.

Glenn Murphy, USA

2608. K. E. Markaki, On the continuous beam having a variable moment of inertia along its length and elastically supported at both ends (in Greek), Techn. Chr. 26, no. 304, 502–509 (Oct. 1949).

The changing moment of inertia of a continuous beam is considered when finding moments at supports in terms of the elastic angles of rotation of the joints and beam in a frame of which beam is a part.

Dimitri Kececioglu, USA

Plates, Disks, Shells, Membranes

(See also Revs. 2623, 2624)

2609. H. Okubo, On the problem of a notched plate of an aeolotropic material, Phil. Mag. 40, no. 308, 913–916 (Sept. 1949).

Paper presents an analytical study of stress distribution due to hyperbolic notches in an orthotropic plate under tension. The stress concentration factor is evaluated numerically for an oak plate. [This problem was solved in Proc. roy. Soc. A 184, 289–300 (1945). Same problem for an isotropic plate was first solved by A. A. Griffith, Rep. Memor. aero. Res. Comm. Lond, no. 1152 (1928).]

A. E. Green, England

2610. O. M. Sapondzhyan, Application of the method of additional reactions to solution of the plate bending problem, plane problem, and problem of torsion of prismatic rods (in Russian), Prikl. Mat. Mekh. 13, no. 5, 501–512 (1949).

Paper contains several examples of solution of boundary-value problems in isotropic elasticity by a procedure termed by author "method of additional reactions." Method depends on ingenious devices suitable to a particular problem under consideration. Essence of idea is to replace consideration of a given boundary-value problem by another one involving region whose boundary, in part, coincides with the original one, and introduce in solution of this new problem sufficient parameters to satisfy the boundary condition of original problem. Idea can be illustrated by problem of small deflections of a thin clamped semicircular plate under a uniform normal load p per unit area of plate. Clamping along diameter of semicircle introduces unknown shearing forces. Now, consider a clamped circular plate of same radius as original one, which is subjected to a load p and to a line distribution of load of intensity q along diameter. If the diameter coincides with y-axis of Cartesian coordinates, and the distribution q is taken in the form $\sum_{n=0}^{\infty} q_{2n} y^{2n}$, formal solution for deflection w is obtained in the form $w(x, y, q_{2n})$ where the q_{2n} are parameters. These parameters can be selected so as to satisfy the condition $\partial w(0, y)/\partial x = 0$. The function w so obtained satisfies the condition of clamping along diameter of semicircle in the original problem.

This device is applied to several plate deflection problems, the torsion problem for a rectangle with a slit, and the plane problem of elasticity for a semicircle fixed rigidly along the diameter and subjected to a uniform radial pressure along the curved boundary.

I. S. Sokolnikoff, USA

2611. S. U. Benscoter, Analysis of a single stiffener on an infinite sheet, J. appl. Mech. 16, 242–246 (Sept. 1949).

Problem treated in paper is determination of stress distribution in an infinite sheet to which is attached a single stiffener of finite length with a concentrated force applied at each end. Solution is carried out as follows: First, formulate an influence function of normal strain in sheet at one point on x-axis due to a concentrated force acting horizontally at another point on x-axis, stiffener being assumed to be on x-axis. Such a function may be obtained from classical solution of two-dimensional stresses in an infinite plate of unit thickness due to a concentrated force at the origin. Physical condition that normal strain in sheet at points along x-axis must be the same as normal strain in stiffener results in an integral equation which corresponds exactly to Prandtl's lifting-line equation for spanwise air-load distribution. The integral equation may be replaced by a system of linear algebraic equations by any of methods known to be applicable to spanwise air-load system. In present paper, Multhopps' method is used to carry out numerical examples with loads on stiffener either both in tension (or compression), or one in tension and the other in compression.

C. T. Wang, USA

2612. Osvaldo Zanaboni, Membrane stresses in cupolas with forces distributed on the surface (in Italian), G. Gen civ. 87, 442–457 (Sept. 1949).

Paper deals with problem of determination of membrane stresses developed in shells of revolution without limitations regarding distribution of external loads. Method of solution differs from the one usually found in textbooks in that equilibrium equations for an element in midsurface of shell are given in Cartesian coordinates instead of intrinsic coordinates of midsurface. This substitution leads to solution of a system of three partial differential equations in three unknown membrane stresses.

A general solution of this system is indicated, several special cases (conical and cylindrical shells, paraboloid, ellipsoid, etc.) are worked out, and formulas for values of membrane stresses at any point in terms of an arbitrary load function are given.

Ernesto Saleme, USA

2613. Phillip Eisenberg, Elastic deformation of thin ellipsoidal shells under hydrodynamic pressures, Taylor Mod. Basin Rep. no. 606, 22 pp. (July 1949).

Report presents a solution for membrane stresses in an ellipsoidal dome of revolution when loaded by external pressures due to an axial fluid stream about the shell. Membrane forces and displacements are calculated, and an estimate of bending stresses is obtained. Author's intended application is to sound dome housing sonar equipment.

Paul F. Chenea, USA

2614. I. N. Vekua, On the theory of elastic shells (in Russian), Doklady Akad. Nauk SSSR 68, 453–455 (Sept. 1949).

The complex stress functions, introduced by Kolosov in 1909, have also been successfully used for other two-dimensional problems of mathematical physics, as for example in the theory of elastic shells. In 1945, making use of this method, author developed a procedure for integrating the equations of spherical shells with arbitrary loads. Now, author extends method to shells of more general forms, making use of isothermal coordinates. A line element ds in such a coordinate system is given by $ds^2 = A(dx^2 + dy^2)$ where A is a positive function. Instead of coordinates x, y, new variables z, ζ are introduced by the relations

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right); \quad \frac{\partial}{\partial \zeta} = \frac{1}{2}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)$$

Equilibrium of forces and moments then furnishes three equations, which are invariant against conformal transformations. In these equations, author introduces components of displacements, so that he finally gets the differential equations of displacements in isothermal coordinates.

Walter Wuest, Germany

2615. **F. Edelman, On the compression of a short cylinder between rough end-blocks,** Quart. appl. Math. 7, 334–337 (Oct. 1949).

Author applies hyper circle method of Synge and Prager [title source, 5, 241–269 (1947)] which places upper and lower bounds on the strain energy of the natural state of a strained elastic member. In this case, specimen is a cylindrical compression block with rough end constraints. Numerical results are given for strain energy in terms of total compressive load F and compression a of cylinder.

D. L. Holl, USA

2616. **V. Z. Vlasov, General theory of shells and its applications in technology,** Gosudarte Izdatel'stvo Tehniko-Teoretičeskoi Liter., Moscow-Leningrad (1949), 784 pp.

Buckling Problems

(See also Rev. 2633)

2617. **B. G. Neal, The lateral instability of yielded mild steel beams of rectangular cross-sections,** Phil. Trans. roy. Soc. Lond. A 242, no. 846, 197–242 (Jan. 1950).

Lateral buckling of deep mild-steel beams of rectangular cross section is studied both theoretically and experimentally for cases in which strains due to bending about the strong axis are in plastic region. Relation is developed between flexural rigidity about weaker principal axis and extent to which beam has yielded under bending about stronger axis. It is shown that torsional rigidity remains constant at its value for elastic torsion. Conditions causing lateral buckling are deduced for pure bending, while a step-by-step procedure is outlined for beams with central concentrated loads and for cantilever beams.

Stress-strain relation is assumed to consist of three straight lines, the first sloping from zero to upper yield point, the second vertical between upper and lower yield points, and the third horizontal at lower yield point. Work-hardening effects are omitted. Curves are given to show theoretical relations developed and influence of ratio of upper and lower yield points.

A testing device is described for applying pure bending to a specimen and, at the same time, allowing unrestrained rotation of ends of specimen about weak axis. Rotation of ends about longitudinal axis is completely restrained. Auxiliary tests were made to determine initial torsional rigidity. Tests on round specimens were included in the studies of effects of bending about one axis on flexural stiffness about the perpendicular axis and on torsional rigidity.

For primary bending stresses well above the yield point, there was good agreement between theoretical and experimental ultimate bending moments.

Marshall Holt, USA

2618. **Chi-Teh Wang, General theory on the buckling of sandwich cylinders,** N.Y. Univ. Sch. aero. Publ. (Aug. 25, 1949).

General theory is formulated for buckling problems of sandwich shells by including effects of large deflection, thus forming a natural extension of the recently published small deflection theory [NACA T.N. 1832 (Mar. 1949)]. This theory is then applied to buckling of a circular sandwich shell under axial compression. Critical stress is found proportional to shear modulus of core material. It is also claimed that critical stress is lower for unsymmetrical buckling than for the symmetrical case.

C. C. Wan, USA

2619. **J. M. Klitchieff, Buckling of continuous beams on elastic supports,** Quart. J. Mech. appl. Math. 2, 257–262 (Sept. 1949).

Author gives an alternative solution for a problem treated by Bubnov in 1912. An elastic beam of constant section and of length L is simply supported at the ends and it is supported by m–l equal elastic springs spaced at equal intervals L/m along the beam. Beam is compressed axially by forces P applied to ends. It is required to find the minimum spring constant for lateral supports which will cause beam to buckle without any deflection of springs. Problem is solved by expressing both lateral deflection of beam and bending moment corresponding to lateral forces applied at intermediate supports in terms of Fourier series, substituting in the differential equation of problem, and solving determinant for buckling into m half waves. After considerable skillful manipulation of resulting infinite sums, author succeeds in obtaining a very simple formula for critical spring constant for any value of m. This spring constant differs less than 3 per cent from values obtained by Bubnov in 1912 (for $m = 2,3,4,5,6,7,9,11$).

Walter Ramberg, USA

2620. **Eric Ingerslev, Lateral stability of I-beams,** Publ. int. Assn. Bridge Struct. Engng. Final Rep., 309–313 (Sept. 1948).

Formulas are presented of an approximate solution for lateral buckling of an I-beam under vertical moment and axial load. Little derivation is given and no comparison with previous solutions or tests.

L. H. Donnell, USA

2621. **Elbridge Z. Stowell and Richard A. Pride, Plastic buckling of extruded composite sections in compression,** Nat. adv. Comm. Aero. tech. Note 1971, 15 pp. (Oct. 1949).

Expressions are derived for the half wave length of buckles and the reduction factor for plasticity η for combinations of plates, such as H- and Z-sections. These expressions contain a parameter, which is a function of tangent and secant moduli, so that η has to be found by trial and error. Results are compared with experimental results for extruded H-section columns of 75S-T aluminum alloy. Correlation is satisfactory, except for high buckling stresses. In all calculations Poisson's ratio for elastic deformations is assumed to be 0.5, as was done in earlier work by principal author.

P. P. Bijlaard, USA

2622. **P. P. Bijlaard, On the torsional and flexural stability of thin-walled open sections,** Nederl. Akad. Wetensch. Proc. 51, no. 3, 314–321 (1948).

Condition of equilibrium of internal and external bending and torsional moments on a column having a thin-walled open section is used as a basis for determining critical compressive load. For a column having a T-section, author shows how to take into account effect of plate buckling deformations as well as flexural and torsional deformations in deriving the stability criteria. A method for applying stability equations in plastic range is indicated.

S. Levy, USA

2623. **L. A. Ringelstetter and C. B. Norris, The effect of a stiffener on the maximum load of flat plywood plates in edgewise compression, with the face grain at 0° and 90° to the load,** For. Prod. Lab. Rep. no. 1553-D, 7 pp. (July 1949).

Paper reports and analyzes some 45 compression panel tests of plywood sheet reinforced by central, rectangular-section, spruce stringers. Loaded edges were clamped, unloaded edges simply supported. Analysis begins by computing L_m which is the sum of experimentally determined maximum load carried by a short column of stiffener, and maximum load which could be carried by two supported skin panels, based upon maximum compressive strength of plywood, its section area and an effectiveness factor allowing for buckling and based upon previous tests reported in same source, no. 1316-I. The ratio of panel-failing load I to load L_m is then plotted against (F_{cu}/P_{cr2}), and lines of constant

$(EI)_s/(EI)_{scr}$ are interpolated. F_{cu} is the compressive strength of plywood given by coupon tests, P_{cr2} is initial buckling stress of panel obtained from a formula given in an earlier report [title source, no. 1555-B], $(EI)_s$ is flexural rigidity of stiffener, and $(EI)_{scr}$ is that value of rigidity which just produces buckling of skins in the two panels as opposed to general buckling involving stringer flexure. This last quantity was obtained experimentally from tests in earlier report (op. cit.). No indication is given of types of failure obtained, and no attempt is made to study influence of stringer slenderness ratio, or ratio of stringer to sheet area.

W. S. Hemp, England

2624. Frederick K. Teichmann and George Gerard, Tests of thin sandwich cylinders under compressive end loads. Part I.—C. C. A. core, N. Y. Univ. Sch. Aero. Publ., 55 pp. (July 15, 1949).

Twelve sandwich cylinders of 24S-T aluminum alloy faces of 0.010-in. thickness and $^1/_8$-in. cellular cellulose-acetate core were tested in axial compression. The radius of the cylinder varied from 10 to 14 in., and the lengths investigated were 12 and 24 in. Circumferential buckling was noticed in cylinders with 10- and 12-in. radius, while wrinkling was noticed in the 14-in. cylinders. There was no apparent length effect on buckling loads. Aspect ratio of buckles was of the order of 3 in circumferential direction to 1 in axial direction. Test results were in reasonable agreement with theoretical values predicted by Gerard (Rev 3, **1673**). Test data are given in appendix, as well as test notes and dimensions of individual specimens.

Conrad C. Wan, USA

2625. J. Brunner, Buckling strength of the upper chord of bridges (in German), Schweiz. Bauztg. 67, 541–543 (1949).

A critical review is presented of the known formula for buckling load of continuously elastically supported columns, as given, for instance, in Swiss stress requirements for steel bridges. By introducing an objectionable assumption, it is shown that this formula should be conservative if the elastic transverse support is discontinuous. Extension of analysis to initial transverse loads, involving initial displacements of supports, is dealt with, as well as the behavior in plastic region.

Since exact theory proves that discontinuity of the supports has an unconservative influence on buckling load, the first part of the investigation is misleading.

H. F. Michielsen, Holland

Joints and Joining Methods

(See also Rev. 2666)

2626. A. R. C. Markl and H. H. George, Fatigue tests on flanged assemblies, Trans. Amer. Soc. mech. Engrs. 72, 77–87 (Jan. 1950).

Cyclically reversed bending tests are carried out on assemblies involving 4-in. flanges of the 300 ASA pressure class. Six commonly used styles of flanges are tested to failure under stresses which produce leakage or cracking within the range from 1000 to 1,000,000 cycles. Tests indicated that even under usually severe bending stresses, flange assemblies did not fail in flange proper; structural failure occurred almost invariably in pipe adjacent to flange. Type of flange is influential in determining endurance strength of assembly, largely by manner in which it affects the stress transfer at joint. A smooth tapering transition afforded by a welding-neck flange provides an endurance strength of assembly equivalent to that of a butt-welded joint between two pieces of pipe. A fillet weld presents a less favorable condition owing to sharp change in cross section and in stress direction. Reduced pipe wall thickness and notch effect caused by threading constitute not only structural weakness, but also threaded flanges are prone to open up under bending and cause leakage. Authors conclude that threaded flanges are unsatisfactory for any but the mildest cyclic services. S-N curves are shown for various assemblies and comparison is made with results of previous flexural fatigue tests of straight pipe. Flanges of types investigated, using approximately 40,000-psi bolt stress, were stiff enough to avoid leakage across gasket and strong enough to prevent failure of flange proper under rated service pressure and superimposed bending stresses of an appreciable magnitude.

T. J. Dolan, USA

2627. Georges Welter, Fatigue tests of spot-welded steel sheets, Weld. Res. Suppl. 14, 414–438 (1949).

An extensive series of fatigue tests under pull-pull loads with spot-welded steel specimens were conducted, and treatments, mechanical as well as thermal, for improving endurance limit were investigated. Microhardness investigations exploring the welded specimens, especially in neighborhood of the spot as well as in the transverse and longitudinal sectons of sheet and spot, were made and used to evaluate roughly stress and strain distribution in sheet. Appearance of easily discernible Lüder's lines on surface of sheet, strained up to plastic range of material, also permitted study of stress distribution.

With aid of an artificial spot under fatigue loads, it was shown that effect of tension stresses combined with bending stresses, localized around the spot, are main factors responsible for failure of thin welded-steel sheets.

Brittle lacquer and strain-gage measurements were used for determination of stress concentration and distribution.

From these tests it could be ascertained that prevention of repeated bending of specimens during fatigue tests seems of primary importance. An increase in moment of inertia in plane of bending, as obtained by embossing, coining, or indenting in and near the spot, helped materially in ameliorating fatigue resistance of thin spot-welded mild steel and stainless-steel sheets.

Article lists test results in detail and is liberally illustrated with photographs, sketches, and diagrams.

Frederick K. Teichmann, USA

2628. J. D. Fast, Causes of porosity in welds, Philips tech. Rev. 11, no. 4, 101–110 (Oct. 1949).

Author attacks problem of porosity in welds from a basic scientific viewpoint with aid of thermodynamic and metallurgical theories. This work has resulted in a fundamental explanation of causes of weld porosity. It is believed this work is of particular value to those interested in the manufacture of welding electrodes.

J. F. Snider, USA

2629. C. W. Mühlenbruch, Analytical and experimental studies of ladle hooks, Iron Steel Engr. 26, 53–71 (Oct. 1949).

Paper reports results of field tests under normal working conditions of high-capacity riveted and welded laminated ladle hooks with regard to stresses and slips between the laminations. Observed stresses are compared with theoretical ones and a few suggestions for design are formulated. Some specimens of riveted and welded joints are also tested for comparison.

A. Hrennikoff, Canada

2630. A. M. Baxter, The distribution of load along nuts, with particular reference to heavy loads, Brit. Shipbuild. Res. Assoc. Rep. no. 3, 5 pp. (June 1947).

Structures

(See also Revs. 2604, 2611, 2623, 2624)

2631. **Henson K. Stephenson and A. A. Jakkula, Highway loads and their effects on highway structures based on traffic data of 1942,** Bull. Agric. Mech. Coll. Texas (5) 8, no. 1, 130 pp. (Jan. 1950).

Report presents an analysis of traffic as to type, size, and weight and gives extensive statistical analyses of data. It should be of major help in planning of design requirements for highways and bridges.

Robert J. Hansen, USA

2632. **J. F. Baker, M. R. Horne, and J. W. Roderick, The behaviour of continuous stanchions,** Proc. roy. Soc. Lond. Ser. A 198, no. 1055, 493–509 (Sept. 1949).

Authors describe tests carried out on small-scale steel stanchions of rectangular and I-section subjected to arrangements of load encountered in building frames. Two main types of loading are distinguished according to whether the stanchion is bent in single or double curvature. A theoretical explanation of results observed is sought by reference to the simple plastic theory, in which it is assumed that sections plane before bending remain plane after bending. Theory of members subjected to combined bending and axial load in partially plastic range is developed and applied to single curvature stanchions. Growth of plastic zones is traced up to stage at which complete plasticity occurs at three sections in stanchion; satisfactory agreement is obtained between theoretical and observed collapse load. When simple plastic theory is applied to double-curvature bending, inaccuracies arise in certain cases due to strain in plastic zones, and the simple theory is therefore elaborated to take account of irreversible nature of plastic strains. Work carried out shows that effect of overstrain is appreciable, but that to ignore it gives a calculated collapse load on the safe side.

K. W. Johansen, Denmark

2633. **Harold E. Wessman and Thomas C. Kavanagh, End restraints on truss members,** Proc. Amer. Soc. civ. Engrs. 75, 951–969 (Sept. 1949).

Paper investigates stability of trusses on basis of known stability criteria (stiffness and series criterion) first introduced by Lundquist (NACA T.N. 617, 1937). All pertinent principles are restated and then applied through extensive use of process of moment distribution. One sample computation is carried out and a number of theorems embodying familiar principles of reciprocity are stated and proved. Paper confines itself throughout to discussion of linear phenomena and of joint rotations without displacements.

E. F. Masur, USA

2634. **Werner Koepcke, Application of the method of moment distribution to symmetric framed structures** (in German), Bauplan. Bautech. 3, 214–216 (July 1949).

Analysis of representative symmetric framed structures is made by moment-distribution method. Resort is had to relations of symmetry and antisymmetry (antimetry) by dealing with but half the structure. When axis of symmetry coincides with one of supporting members, girder coming into that support is cut and treated as fixed-ended at plane of symmetry of structure and load. If load is antisymmetric, however, effect of support must be considered and the distribution factors for ends of girders adjacent to plane of structural symmetry are shown to depend upon number of supporting columns at that point. If one vertical column having stiffness factor S_c is used at plane of symmetry, the distribution factor at its junction with girders is $S_c/(2S_g + S_c)$ in column, and $2S_g/(2S_g + S_c)$ in girder on half structure under consideration. If two inclined columns are used, the corresponding factors are $S_c/(S_g + S_c)$ and $S_g/(S_g + S_c)$.

When axis of structural symmetry passes through middle of a bay, the stiffness factors for that bay may be taken as 0.50 EI/L under symmetrical load, as 1.50 EI/L under antisymmetric, L being full span length in each instance. Except for emphasizing use of these factors, described procedure is that in common use in 1943 when paper was originally prepared.

J. S. Newell, USA

2635. **L. T. Wyly, M. B. Scott, L. B. McCammon and C. W. Lindner, A study of the behavior of floor beam hangers,** Bull. Amer. Rly. engng. Assoc. 51, no. 482, 51–73 (Sept.-Oct. 1949).

Effects of the form of hanger sections and of floor beam and stringer deflection upon stress distribution under static and dynamic loads were investigated on the single track, 127-ft. through truss span of the Illinois Central R.R. near Galena, Ill. Each hanger consisted of two 12-in., 30-lb channels connected by occasional tie plates. Strains were measured using SR-4 gages attached at 11 sections in each hanger in one end span. Various classes of locomotives were used in test program, but report presents only results obtained from loadings with class 2800 locomotives (2-10-2). In static tests, third driver was located over floor beam between hangers. Dynamic tests were made at speeds from 4 to 57 mph.

From test data, it is concluded that the diaphragm between the channels was not sufficient to transfer half of load to outer channel. In places, inside channel carried 50 per cent more load than outside channel. A plane section did not remain plane, each channel tended to act independently, and maximum bending stresses were greater than axial stresses. Planar stress distribution was attained in individual channels except at rivet holes. Tie plates did not form satisfactory bracing as their use resulted in severe local bending.

Dynamic loading resulted in same type of stress distribution as did static loadings, and all total impact loads carried by hangers were within present AREA specifications. However, at a speed of 43.4 mph, maximum stresses were as much as 2.75 times the average stresses. Dynamic stresses at engine speeds of 5 mph agreed closely with static stresses. A rational design of this type of hanger does not appear practicable.

Glenn Murphy, USA

2636. **I. G. Hannemann, Distribution of moments and additional moments in arch bridges with rigid arch or rigid stiffening girder** (in Danish), Bygnstat. Medd. 19, no. 1, 1–23 (1948).

2637. **K. W. Johansen, The ultimate strength of reinforced concrete slabs,** Publ. int. Assn. Bridge Struct. Engng. Final Rep., 565–570 (Sept. 1948).

An outline of the theory of lines of fracture of reinforced concrete slabs is given. Through the geometric conditions, since the elastic deformations are insignificant as compared with the plastic ones, the shape of the fracture can be determined. The moments along the fracture being a maximum, the shear and the twisting moment become zero, except at a free edge. The equations of equilibrium for the fractured parts can be set up and the shape of the fracture and breaking moment can be determined. A simple method of approximation can be developed using the principle of virtual work. Method is illustrated by examples.

From author's summary by Andrew Brodsky, USA

2638. **P. Haller, The buckling strength of masonry of manufactured materials** (in German), Schweiz. Bauztg. 67, no. 38, 531–536 (Sept. 17, 1949).

An experimental investigation is reported of the buckling strength under compression of various types of wall construction

for various slenderness ratios and load eccentricities. Recommended design factors of safety are given.

H. G. Hopkins, England

2639. A. H. A. Hogg, Pavement slabs on non-rigid foundations, Proc. sec. int. Cong. Soil Mech. Found. III, 70–74 (1948).

Curves are given showing stresses and displacements for a thin slab resting on an isotropic elastic layer of finite depth and carrying a single concentrated load. Behavior of this system is compared with that to be expected from a slab resting on a nonrigid foundation, and it is shown that a fairly satisfactory agreement can be found with such experimental results as have been published if properties of the subgrade are represented by an equivalent elastic layer.

From author's summary

2640. B. G. Heebink and A. A. Mohaupt, Effect of defects on strength of aircraft-type sandwich panels, For. Prod. Lab. Rep. 1809, 16 pp. (Sept. 1949).

Strength tests were performed on panels in edgewise compression, and also with tension normal to the surface, to determine effect of numerous manufacturing defects. Tests were made on five types of construction employing aluminum or glass-cloth facings with several types of cores.

Stanley U. Benscoter, USA

2641. L. F. Coffin, Jr., P. R. Shepler, and G. S. Cherniak, Primary creep in the design of internal-pressure vessels, J. appl. Mech. 16, 229–241 (Sept. 1949).

Normal basis for design of high-temperature high-pressure vessels is to consider only conditions of secondary creep. In design of such vessels that are to have a relatively short life, initially rapid primary creep should be considered together with changes of stresses caused by plastic flow. An evaluation is given of stresses and permanent strains resulting from loading a thick-walled cylinder with constant pressure for a given time period when primary creep is considered. Results are compared with calculations considering only secondary creep. For a particular chromium-steel cylinder operating under 12,000 psi at 850 F and having a wall ratio of 2, permanent strain at end of 25 hr as determined by primary creep method was found equal to that at end of 2000 hr by secondary creep method. It is concluded that, in design of pressure vessels for short life, considerations of elastic conditions and primary creep are essential, while for long-life considerations, secondary creep analysis is sufficient.

Irwin Vigness, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 2600, 2655, 2677)

2642. Fernand Charron, Flow of plastic bodies (in French), C. R. Acad. Sci. Paris 230, 614–615 (1950).

A viscoplastic body is considered with a relationship between the shear stress τ, and shear-strain rate γ, of the form $\tau = f + \eta\gamma$ where f is a friction coefficient, and η a viscosity coefficient. Steady flow down a circular cylindrical pipe produces a rigid core, the corresponding velocity distribution is determined, and an equation giving the discharge in terms of the pressure gradient is established. The simpler limiting form when the pressure gradient is only slightly greater than that required to start the flow is also given.

E. H. Lee, USA

2643. V. P. Degtyarev, On the question of strength and cold brittleness in presence of initial static stresses (in Russian), Zh. tekh. Fiz. 19, no. 8, 882–891 (Aug. 1949).

Four steels with medium carbon content, and cast iron with 2.61C, .5 Mn and 1.82 Si were subjected to Charpy notch-bar impact test with and without applied compression or torsion stress. A sample of steel with .87 C shows an increase of impact strength from 2 to 7 m-kg/cm² with increase of compression stress up to 40 mg/mm², while impact strength of another steel with .55 C increases from 5.5 up to 7.2 m-kg/cm² with increase of compression stress up to 22 kg/mm². Temperature range of beginning cold brittleness of former steel decreases from −10 to −20 C, to −40 to −60 C. A torsion stress of 16 kg/mm² increases impact strength of second steel from 7 to 10.5 m-kg/cm², while another steel with .53 C shows an increase of impact strength from 7.5 to 9 m-kg/cm² with torsion stress of 17 kg/mm² and a decrease of temperature range of beginning cold brittleness from −70 to −80 C to −80 to −100 C.

With cast iron, a small decrease of notch-bar impact strength has been found, but no change without a notch.

Georg Masing, Germany

2644. Charles Goodeve, The mechanism of thixotropic and plastic flow, Proc. int. rheolog. Congr. Holland, II: 5–11; III: 18–20 (1948).

The term "thixotropy" is used in a generalized sense to include "Bingham material" (finite yield stress; shear-strain rate a linear function of shear stress). Thixotropy in usual narrower sense is distinguished by a hysteresis in reestablishing yield stress after deformation.

It is assumed that thixotropic materials contain long molecules or long coherent chains of structural elements which are strained and broken during flow; they are at the same time currently reformed. Hysteresis in latter process leads to thixotropy in narrower sense. Yield stress is the number of such chains per unit area of a layer whose thickness is the length of the chains multiplied by the mean force in chains which is half of breaking force. This relationship is derived by means of momentum theorem of mechanics.

E. Orowan, England

2645. N. Thorley, The calculation of the activation energies of recovery and recrystallization from hardness measurements on copper, J. Inst. Metals 77, part 2, 141–161 (1950).

Changes which take place in crystallites of a metal when it is cold worked and then annealed are reviewed, and suggestions are made for observing these changes directly, using x-ray line-broadening measurements, in an investigation of fundamental processes of annealing.

Cook and Richards' two-stage theory of recrystallization for an isothermal anneal [J. Inst. Metals 73, p. 1 (1947)] is extended, and a new equation obtained which fits their experimental results for the hardness of copper better than previous equations. It is found that the process followed by hardness measurements is one in which strain release and recrystallization both begin at zero time and proceed simultaneously in different parts of the specimen until annealing is complete.

Experimental results of Cook and Richards on hardness of copper are analyzed in some detail, and a direct method is developed whereby activation energies of both recovery and recrystallization processes are obtained separately. Calculated values are of same order, so that recovery and crystallization may be the same energetically and, in this sense alone, the whole anneal could be described as a single-stage process. Physically, there would still be two manifestations of this single activation energy, namely, removal of strain, and growth of crystallites from freshly formed nuclei.

From author's summary

2646. Doris Kuhlmann and Georg Masing, Investigations on plastic deformation of copper wire (in German), Z. Metallk. 39, 361–375 (Dec. 1948).

Isothermal inelastic deformation of commercially pure copper

wire, from soft to hard drawn, tested in the shape of helical springs under constant tensile loads, is observed as function of time, of applied stress, and of various heat treatments. Wire diameter varied between 0.03 in. and 0.04 in., spring diameter between 0.185 in. and 0.230 in., the 0.2 per cent yield stress between 5600 psi for annealed wire to 60,000 psi for hard-drawn wire. Springs were loaded immersed in distilled water; testing temperatures, which varied between 15C and 41C, were controlled by variation of water temperature. Observed creep curves are discussed in light of Becker-Orowan theory of slip, and of dislocation concept of Taylor; they are also compared with Andrade's known creep tests on polycrystalline wires and some Russian tests on creep of single crystals. While expected linear relation between log of creep rate and inverse Kelvin temperature is confirmed, computation of the "activation energy" gives inconsistent results. Authors find their results do not confirm the Becker-Orowan equation. For the strain-time function in torsion, they propose a power law of the form $\gamma = \gamma_0 + Ct^\alpha + \Delta\gamma$, where γ denotes total shear strain, γ_0 elastic component, t time, α and C are constants, and $\Delta\gamma$ the so-called spontaneous (irrecoverable) strain occurring on load application. While this part of strain is attributed to creation of new dislocations, the time-dependent flow is attributed to their gradual dissolution under stress.

A. M. Freudenthal, USA

2647. **Robert Sauer, On the glide curves of plane plastic stress distributions with arbitrary law of plastic flow** (in German), Z. angew. Math. Mech. 29, 274–279 (Sept. 1949).

For a plane state of stress, stress components may be derived from a potential function. A transformed linear potential equation is obtained by a Legendre transformation. The characteristics C_1, C_2 of this equation are related to the glide curves G_1, G_2 in such a way that the tangent at a point P of a parameter line in one net is parallel to tangent of other parameter line in other net in the correlated point P' (i.e., the tangent along C_1 is parallel to the tangent along G_2 in correlated points). The C-nets may be derived for an arbitrary law of flow, and hence the glide lines may be constructed. For the maximum shear stress law, results are discussed in details. Analogous relations for the Mach nets in plane supersonic irrotational flow are mentioned.

Albert Kochendörfer, Germany

2648. **Rodney Hill, General features of plastic-elastic problems as exemplified by some particular solutions,** J. appl. Mech. 16, 295–300 (Sept. 1949).

Complete solutions of several plastic elastic problems are found, based on the Reuss equations $d\epsilon_{ij} = d\sigma_{ij}/2G + \sigma'_{ij}\, d\lambda$ and $d\epsilon_{ii} = (1 - 2\nu/E)d\sigma_{ii}$; in tensor form, where σ_{ij} is stress, σ'_{ij} is deviatoric part of the stress, ϵ_{ij} is natural strain, λ is a proportionality function, and E, G, and ν are the elastic constants [see ZAMM 10, p. 266 (1930)]. These problems are: (1) Uniform extension under condition of plane strain; (2) expansion of a spherical shell (or hole in an infinite medium) by internal pressure; (3) the same for hollow cylinder; and (4) torsion of prismatic bar. One purpose in getting these solutions was to investigate permissibility of certain simplifications. Author concludes that when plastic region is completely enclosed by elastic material with small displacements, it is usually necessary to consider the effect of elastic strains within the plastic region (first term on right side of first Reuss equation). However, when large strains occur over a considerable part of plastic region, it may be permissible to neglect elastic components of strain within it (equivalent to replacing the Reuss equations by Levy-von Mises relations. If plastic zone is not completely enclosed by elastic material, as in penetration by a punch, for example, effect of neglecting elastic strains in plastic zone is still smaller. When entire body under consideration becomes plastic, effect of elastic components is negligible after a small deformation has occurred. Author also points out that for a complete determination of all variables, whole history of deformation must be found, but that under some conditions certain quantities can be solved without a complete analysis. This may be the case when material does not harden and strains are so small that changes of boundary can be neglected.

Merit P. White, USA

2649. **A. McCance, The plastic behavior of solids,** J. Iron and Steel Inst. 163, part 3, 241–249 (1949).

The views of some workers that the plastic behavior of solids can be formulated by an extension of elasticity theory is regarded as untenable by author. It is assumed that atoms in a metal can exist in two states: normal state with a regular lattice, and cold-worked state with a distorted lattice. With each of these states is associated a specific volume so that change in specific volume after cold work reflects proportion of atoms which have changed from one state to the other. A general equation: $S = 2S_0 e^{-b \cdot a} \sinh a(b^2 - c^2)^{1/2}$ is developed, in which "true" stress S is expressed as a function of the contraction in area a, a constant S_0, and the coefficients b and c. The terms are evaluated by curve fitting for a number of metals. It is concluded that work hardening is due to an increase in contribution of surface tension forces arising from creation of new surfaces at slip planes. In terms of coefficients of his general equation, author discusses brittle behavior and effects of temperature, pressure, and atomic structure on behavior of a number of metals, rock salt, marble, starch, and rubber. Based on assumption that creep is solely a result of diffusion processes, an equation is proposed for metals expressing creep life to rupture under any given stress.

T. J. Dolan, USA

2650. **R. Hill, The theory of plane plastic strain for anisotropic metals,** Proc. roy. Soc. Lond. Ser. A 198, 428–437 (1949).

Yield criterion and plastic stress-strain relations, generally formulated in author's recent paper [Proc. roy. Soc. Lond. Ser. A 193, p. 281 (1948)], are discussed in detail for an anisotropic two-dimensional strain state which is supposed to be uniformly distributed over the material. The equations are hyperbolic; the characteristics coincide with the directions of maximum-shear-strain rate; variation of stresses along these is expressed in terms of elliptic functions. Geometrical properties of field of characteristics, which are similar to those in isotropic strain states [H. Hencky, Z. angew. Math. Mech. 3, p. 241 (1923)], are established. Theory is applied to problem of indentation by a flat die.

Albert Kochendörfer, Germany

2651. **L. D. Sokolov, On the influence of velocity of deformation and temperature on the resistance of metals to plastic deformation** (in Russian), Doklady Akad. Nauk SSSR 67, 459–462 (July 1949).

From over 1000 true stress-strain curves (in compression) of Sn, Pb, Al, Zn, Cu, a brass, and 10 steels, author concludes that Ludwik formula $Y = Y_0 + k \log v$ (Y yield stress, v rate of deformation) gives a good approximation only for high-melting metals at not too high temperatures. In entire range of temperatures and speeds (up to 1200 C for steels), a better fit is obtained with Davidenkov-Wittmann-Stepanov formula $\log Y/Y_0 = N \log v/v_0$, where $N = m\, T/T_m$ (m a constant, T_m temperature of melting).

Reviewer's remark: The parabolic formula is an attempt to take into account recovery (thermal softening). Recent results show that at least three phenomena are involved in the velocity- and temperature-dependence of the yield stress (recovery, tran-

sient flow, and the velocity-and temperature-dependence of strain hardening), so that above formulas cannot be expressions of a physical law. E. Orowan, England

2652. Franco Levi, On the evaluation of the effect of retarded elasticity in hyperstatic solids (in Italian), G. Gen. civ. 87, 343–350 (1949).

Author continues his previous studies of plastic phenomena with an investigation of the stress-strain characteristics of an axially loaded member composed of longitudinal elastic elements and elastoviscous elements. The stress-strain behavior of elastoviscous elements is assumed to follow law $\epsilon = \sigma/E + {}_0\int^t \sigma \times (d\epsilon_0/dt)dt$. Reported cases are: (1) Constant distortion (relaxation) when the two materials have same modulus of elasticity, (2) variable distortion with same moduli, and (3) deformation when materials have different moduli. Behavior of these solids is compared with that of viscous materials.

Glenn Murphy, USA

2653. Joseph Marin, Stress-strain relations in the plastic range for biaxial stresses, J. Franklin Inst. 248, 231–249 (Sept. 1949).

Extensive tests have been made of stress-strain relations of aluminum-alloy tubes under a wide range of biaxial stress ratios. In conformity with prior investigations of this type, it is found that significant stress vs. significant strain curves are identical for all axial ratios, apart from the termination point. Ductility at fracture was more nearly constant when expressed in terms of significant strain than when expressed in terms of nominal strain. Since biaxial ratio remained constant during tests, no comparison could be made between the various deformation theories.

Clarence Zener, USA

2654. Doris Kuhlmann, Georg Masing, and Joseph Raffelsieper, On relaxation theory (in German), Z. Metallk. 40, 241–246 (July 1949).

Using a combination of dislocation and chemical reaction-rate theories, recovery of a single crystal from slight plastic deformation is calculated as a function of time and of temperature. Results show fair agreement with experimental work on single crystals of aluminum at temperatures ranging from 100 to 400 C, and for times ranging from 1 to 6000 minutes. Edward Saibel, USA

Failure, Mechanics of Solid State

(See also Revs. 2653, 2664)

2655. G. V. Uzhik, On the principles of the theory of strength and plasticity (in Russian), Izvestiya, no. 10, 1433–1455 (Oct. 1949).

Rediscovery of Ludwik-Davidenkov scheme of mechanical behavior of ductile metals (shear stress condition for plastic deformation, tensile stress condition for brittle fracture). Ductile fracture, identified with shear fracture, is believed to obey a critical shear stress (i.e., a critical shear strain) condition. Remark by reviewer: If this were so, maximum strains obtainable by stretching and by rolling would be approximately equal.

E. Orowan, England

2656. G. V. Uzhik, Theory of the load-carrying capacity of a metal (in Russian), Doklady Akad. Nauk SSSR 68, no. 1, 61–64 (Sept. 1949).

Author shows analytically that ultimate load P_p carried by a tensile specimen with a deep circumferential groove may be substantially increased by appropriate mechanical working of the outer layer of metal in the area of stress concentration. Based on his previously published theoretical development (see preceding review), author presents a family of theoretical load curves for a specimen of 18-mm diam having a V-groove 4-mm deep. Without indicating how this is to be done, author shows that if shear strength of the material in an outer layer of thickness t is lowered by working this layer, for small t (to about $0.03a$, where $2a$ is specimen diameter at groove) P_p drops, but with increased depth of working P_p increases until at the optimum point ($t = 0.40a$, shear strength ratio, worked to unworked metal, $= 0.322$), P_p is doubled. This effect is the more pronounced the closer to unity is the ratio of tensile to shear strength in unworked material. This ratio was 1.5 in given example. Experimental results are not given, but author indicates that experiment substantiates above theory. Walter W. Soroka, USA

2657. M. Roš and A. Eichinger, The danger of rupture in solids under static loads (in German), Eidgenöss. MatPrüfAnst. Ber. no. 172, 118 pp. (Sept. 1949).

This is one of the most important treatises ever published on the subject and develops several papers by same authors published in last 25 years. Authors start by reviewing thoroughly fundamental elastic concepts and definitions related to solid behavior, go later through plasticity considerations, and deal with several practical analyses of flow conditions. The different types of fracture and theories explaining this phenomenon are reviewed with numerous references to tests conducted by investigators all over the world. Large use is made here of concept of octahedral stresses and of "reference" stresses based on octahedral stresses, and great effort is made to apply same type of equation used in elastic field to plastic deformations. Authors study many of their numerous tests in light of their theoretical considerations and get conclusions on the way of fracture of several metallic and nonmetallic materials. They emphasize that to each material corresponds a theory of failure, function of its own structure and of phenomena influencing its deformation. They do not seem however to be interested in size effects, statistical considerations in failure, and the Griffith theory. Authors consider this volume as an introduction to three more volumes to be published, dealing with fatigue, impact, and stability. Almost 400 photographs and diagrams are included. A. J. Durelli, USA

Material Test Techniques

(See also Rev. 2673)

2658. Irving A. Denison and Melvin Romanoff, Soil-corrosion studies, 1946: Ferrous metals and alloys, J. Res. nat. Bur. Stands. 44, 47–76 (Jan. 1950).

Report contains results of measurements of corrosion made on a variety of wrought and cast ferrous materials after exposure to different soil conditions for periods up to 14 years. The steels ranged in composition from fractional percentages of nickel and chromium to high concentrations typical of wholly austenitic steels. Soils ranged from well-aerated soils deficient in soluble salts to poorly aerated soils containing high concentrations of water-soluble materials. Magnitude and progress of corrosion with respect to both weight loss and pitting are interpreted in relation to properties of soils at the test sites. Comparison is made of corrosion of specimens of plain wrought materials and cast iron. From authors' summary by T. J. Dolan, USA

2659. R. Cabarat, A new dynamic method for measuring elasticity constants (in French), Rev. Metall. 46, 617–621 (1949).

A precision method is described for measuring Young's modulus or the internal damping of a material as a function of temperature

Test specimen in form of a small rod is mounted at its midpoint, one end forming one plate of a condenser in a tuned oscillating circuit. Electrostatic force causes rod to vibrate lengthwise when circuit is tuned to natural frequency of rod. Nonconducting specimens can be tested by coating them with a thin conducting film, such as a metallic film evaporated onto specimen. Specimen and a thermocouple can be placed in an electric furnace, so that measurements at high temperature are feasible. Error of the method is stated to be on the order of 0.3 per cent.

Curves of Young's modulus are presented for a series of nickel-iron alloys at temperatures up to 500 C. Curves of amplitude versus frequency are given for glass and Plexiglas. From width of resonance peak at half power the logarithmic decrement can be computed.

This method has been used in the U.S.A. for low-temperature measurements [P. G. Bordoni, Study of the behavior of solids at low temperatures by vibration measurements, M.I.T. Acoustics Lab. quar. Prog. Rep. (July-Sept and Oct.-Dec. 1948)].

Brandon G. Rightmire, USA

2660. **Donald C. Buffum, Investigation of square sub-sized v-notched Charpy specimens,** Amer. Soc. test. Mat. Bull. no. 160, 45-47 (Sept. 1949).

Impact tests on one steel with a single heat treatment are reported for subsized V-notched Charpy specimens examined over a wide range of temperatures. Transition from ductile to brittle fracture is reported on basis of both total energy and per cent fibrous fracture. Variation in specimen size includes reducing the length, cross section, notch depth, and notch curvature in different combinations. Although a wide range in transition temperature is found for various specimen size, least variation is observed when full fibrous fracture is used as criterion for defining transition temperature. Other observed results show a general reduction in transition temperature with reduced specimen cross section, negligible notch radius effects, and lowered transition temperatures with shallow notches.

Louis F. Coffin, Jr., USA

2661. **Rudolph C. Valor, Jr., Volume changes observed in small concrete cylinders during freezing and thawing using a mercury displacement dilatometer,** J. Res. nat. Bur. Stands. 43, 1-27 (July 1949).

Dilatometer is described in detail. The specimens (1.92-in. diam and 7.20-in. height) were molded, using a normal Portland cement and sand and gravel of Potomac River origin in the proportions 1 cement: 2.8 sand: 2.5 gravel, by weight. One half of the specimens were molded of the basic mix, and the other half had an air-entraining agent added. Numerous variations in curing, in conditioning, and in cycling were introduced. All cycles ranged from +40 F to −20 F. A fast cycle was of 1-hr duration; slow cycles, 32 hr or more. Moisture condition of specimens ranged from air-dry to vacuum-saturated.

Author found the volume changes for all specimens in air-dry condition to be directly proportional to changes in temperature, but this was not the case for partially saturated specimens. Departures were attributed to freezing of water within pores of concrete. He found most specimens showed a residual expansion at completion of a cycle, which for repeated cycles was cumulative. Vacuum-saturated specimens were found to yield excessive residual expansion after only one cycle.

Frank L. Mehringer, USA

2662. **H. Majors, Jr., B. D. Mills, Jr., and C. W. MacGregor, Fatigue under combined pulsating stresses,** J. appl. Mech. 16, 269-276 (Sept. 1949).

Article is concerned with tests of specimens under combined tangential and longitudinal stresses. Apparatus used for making tests is described, and some test results obtained for annealed SAE 1020 steel are given.

Combined-stress pulsator delivers a pulsating load to specimen by means of oil from a fuel-injection pump. Test piece is a carefully machined hollow tubular specimen with a thin wall. Method of stressing is by oil pressure from within. Ratio of longitudinal and tangential stresses is controlled by providing suitably designed heads in upper ends of tubular specimens. These are so machined as to direct the pulse and flow of oil to obtain needed results. Ratios with tension and compression are possible with this system. Description is provided in article. Stresses actually obtained, both static and dynamic, were measured by means of resistance-wire strain gages. Mechanical properties of material used in tests were carefully determined, as was the uniaxial fatigue strength.

Results are given in detail and S-N curves are shown. There is a brief summary of various theoretical interpretations of uniaxial fatigue stresses applied to biaxial cases. Conclusion is drawn that of possible theories listed, actual results of tests described in article agree most closely with distortion-energy theory.

Andrew N. Zamboky, USA

Mechanical Properties of Specific Materials

(See also Revs. 2651, 2654, 2656, 2661)

2663. **W. Rostoker, The effect of applied load in micro-indentation tests,** J. inst. Met. 77, part II, 175-184 (1950).

Among the many mechanical tests of materials, determination of hardness is particularly important because it enables very small regions of a larger surface to be tested and their strength to be estimated without destruction of specimen. Author has carried out such micro-indentation tests on a number of polished metals which had been cold worked to various degrees, using a Bergman diamond pyramid indenter with static loads varying from 25 to 200 gm applied for 30 sec. It was found that the hardness number (based on surface area of impression) increases not only with amount of previous cold work, as would be expected, but also with applied load, which appears to be due to work hardening during, and elastic recovery after, test [see Tabor, Proc. roy. Soc. A 192, p. 247 (1948)].

For the same reasons, the Meyer hardness (based on projected area of impression) increases similarly. Author shows that Meyer index varied from about 2 for materials in annealed state to about 3 for 50 per cent reduction in area for all tests, and suggests that this parameter may be taken as an indication of capacity of material for cold work. Value of paper would have been enhanced considerably if full details of physical properties, particularly the stress-strain relationships up to failure, had been given for tested materials, so that results of hardness tests could have been analyzed on their basis. This would have enabled a more fundamental approach to problem of estimating strength of materials from their hardness.

G. G. Meyerhof, England

2664. **A. B. Bagsar, Notch sensitivity of mild steel plates,** Weld. Res. Suppl. 14, 484-506 (Oct. 1949).

Notch-sensitivity characteristics of several open-hearth, rimmed, semikilled, and killed steels of structural and pressure-vessel qualities, in form of plates and representing thickness range of 1/4 to 2 1/4 in., are evaluated by use of several types of notched bar tests. Tests used are cleavage-tear group consisting of tensile-bend, tensile, and bend tests, and impact group consisting of a series of Charpy tests using keyhole and V-notched coupons. Criterion of notch sensitivity used in cleavage-tear tests is the

changes in nominal yield and nominal breaking strengths, most of which are determined cinematographically, and that in impact tests is the change in energy of fracturing.

Cleavage-tear tests are found to have certain inherent advantages over impact tests, including the following: Cleavage-tear tests are more informative regarding notch sensitivity characteristics and determine the significant temperature T_n at which yield and breaking strengths of a steel coincide with each other; they disclose anisotropic properties of steel plates; they evaluate effect of section thickness; they reveal which heat treatments increase and which ones decrease resistance of a steel to cleavage crack propagation; and they are simpler and more economical. Although no definite relationship was established between grain size and transition temperature, killed steels as a group were found to be superior to other grades of same thickness.

Frederick Seitz, USA

2665. A. Pomp and A. Krisch, Tension and impact tests with chromium-nickel-molybdenum, chromium-molybdenum and chromium-vanadium-steel alloys at high and low temperatures (in German), Arch. Eisenhüttenw. 20, no. 9/10, 323–328 (Sept./Oct. 1949).

Authors carried out tensile tests in temperature range of −50 to 60 C and notch-impact tests in temperature range of −50 to 250 C on Cr-V steels (C 0.3–0.5, Cr 1.4–2.8, V 0.05–0.3), Cr-Mo steels (C 0.3–0.45, Cr 2–2.5, Mo 0.3–0.5) and Cr-Ni-Mo steels (C 0.35, Cr 1–1.5, Mo 0.3–0.5, Ni 1–2). Test specimens were taken from forged and tempered bars (length 250 cm, diam 19.5 to 26.5 cm) in transverse direction and for tests at room temperature also in longitudinal direction.

Tensile strength σ_B showed little difference for various groups of steels as well as for different directions. For each group separately the yield stress $\sigma_{0.2}$ was also almost the same in both directions. Ratio $\sigma_{0.2}/\sigma_B$ varied from 0.73 for Cr-Ni-Mo steels to 0.83 for Cr-V steels. Elongation δ, reduction of area ψ, and the impact value K were on the whole the same for all groups but showed in given sequence an increased spread and difference in both directions, giving as mean values δ_t/δ_l 0.86, ψ_t/ψ_l 0.79, K_t/K_l 0.62.

For all groups σ_B decreased from 110 per cent of room temperature value at −50 C to 90 per cent of that value at 200 C, increased to a maximum at 300 C equal to room temperature value (or less), and decreased again to 45 per cent at 600 C. $\sigma_{0.2}$ decreased continuously to approximately 20 per cent of room temperature value at 600 C. Ratio $\sigma_{0.2}/\sigma_B$ was 0.78 at −50, 0.81 at 100 C, and 0.37 at 600 C. δ decreased to 80 per cent of room temperature value at 200 and increased to 22. 5 per cent at 600 C. ψ increased from 90 per cent of the room temperature value at −50 C to room temperature, decreased to 85 per cent at 300 C and increased again to 160 per cent at 600 C. The impact value was almost constant from +50 C to 250 C. Below +50 C there was a continuous decrease and in some cases cleavage fracture occurred at −50 C.

J. H. Palm, Holland

2666. W. I. Pumphrey, The welding of aluminum alloys, Metallurgia 40, 239–245 (Sept. 1949).

Some aluminum alloys crack at temperatures above the solidus during welding or casting operations, others during cooling of the molten metal. Paper, essentially a progress report, draws upon fifteen articles directed toward determining factors that affect cracking during either operation. It shows that qualitative indications of crack-resisting characteristics of an aluminum-silicon alloy may be had by determining length of cracks in a torus cast from alloy, or in a welded joint made in sheet stock so held that weld is strained as metal cools. These indications are correlated with phase diagram for the alloy.

The cast-ring and restrained-weld procedures are discussed critically, and it is concluded that, though not highly precise, their accuracy suffices for explorative phases of investigation. Curves show effect on cracking of changes in silicon content of an aluminum-silicon alloy, or of changes in either silicon or copper content of an aluminum-silicon-copper alloy. Suggestions and observations scattered through paper appear to have greater value than material in "Conclusions." Joseph S. Newell, USA

2667. Erich Meyer-Rässler, Strength properties of various press-casting materials (in German), Z. Metallk. 40, no. 9, 355–358 (Sept. 1949).

Paper is concerned with investigation of strength properties of various pressure-die-casting materials, available in form of test pieces, according to DIN E 1725. Test bars were accurately manufactured. About 300 bars from three aluminum alloys, four magnesium alloys and one brass alloy were investigated. Hardness, 0.2 per cent yield limit, tensile strength, and strain are plotted in frequency diagrams. For comparison, some tests on zinc alloys were performed. It was found that strength properties of the zinc alloys were best, followed respectively by the aluminum alloys, brass and magnesium alloys.

From author's summary by J. H. Palm

2668. P. Bardenheuer and W. A. Fischer, The mechanical properties of titanium alloy sheets cooled in air after the rolling process (in German), Arch. Eisenhüttenw. 20, no. 9/10, 313–322 (Sept./Oct. 1949).

Three kinds of alloys were produced with increasing titanium contents from 0.4 to 1.2 per cent and constant carbon content of about 0.10 per cent. These alloys were then rolled to form sheets with thickness being 20 and 12 mm at different rolling temperatures. Another alloy with same carbon content and a titanium content of about 0.6 per cent was produced in a Siemens-Martin furnace and was again rolled to form sheets. Mechanical properties of those sheets were then investigated at room temperature, 350 and 500 °. Endurance limits of those sheets were also tested at 500 ° according to DIN-Vornorm DVM 117/118. Result appears that alloy produced from arc furnace with 0.69 per cent C, 0.30 per cent Si, 0.58 per cent Mn and 0.66 per cent Ti has the most favorable mechanical properties and best value of endurance limit. In spite of these good mechanical properties, however, titanium-alloy sheets cannot be used extensively until they can be produced without defect. Hence, a better knowledge of metallurgical behavior of titanium in steel alloy is necessary.

R. G. Sturm, USA

2669. A. E. Johnson, The creep of a nominally isotropic aluminum alloy under combined stress systems at elevated temperatures, Metallurgia 40, 125–139 (July 1949).

Author reports an investigation of creep properties of a cast-aluminum alloy (Hidiminium RR 59) at temperatures of 150 and 200 C under complex plane stress systems. Report begins with an introduction, including a brief summary, of theories for creep under complex stress system, with special reference to the work done by W. Prager, and a short description of previous experimental work in this field. From the 150-hr creep tests on initially isotropic aluminum alloy at 150 and 200 C, author draws following conclusions: At both temperatures, material behaves isotropically at low and moderate stresses, but at combined higher stresses at 200 C, material shows some anisotropy. (Combined tests at higher stresses were not made at 150 C.) The creep rate-stress relation for this material at 200 C could be represented by an equation of the type

$$C_1 = \{A[(\sigma_1 - \sigma_2)^2]^{m_1} + B[(\sigma_1 - \sigma_2)^2]^{m_2}\}\{(\sigma_1 - \sigma_2) - (\sigma_3 - \sigma_1)$$

where C_1 is creep rate in direction of major principal stress, σ_1, σ_2, σ_3 principal stresses, and m_1, m_2, A, B constants. In the case of lower and moderate stresses, it is sufficient to use only the first term in the first bracket. Similar equations are valid for 150 C.

Values of the constants for this special material are given for both temperatures, together with charts of major principal stress vs. major principal creep rate for various values of the minor principal stress, and charts showing major principal stressses giving various specific creep rates at 150-hr tests for several ratios of minor to major principal stresses. Finally, author states that if a material obeys the Hencky criterion of plastic strain and exhibits a linear log-stress vs. log-creep-rate relation in pure tension or pure torsion creep, it is possible to determine creep rates under combined plane stress systems from pure tensile and torsion creep characteristics. Sven Edling, Sweden

2670. **John J. Lamb, Isabelle Albrecht, and Benjamin Axilrod, Mechanical properties of laminated plastics at −70°, 77°, and 200°F,** J. Res. nat. Bur. Stands. 43, 3, 257–289 (Sept. 1949).

A number of laminated plastics of various makes, consisting of an unsaturated polyester reinforced with glass fabric and of phenolic resins reinforced with asbestos fabric, high-strength paper, cotton fabric, and rayon fabric were tested in impact, bending, tension, and compression at various temperatures between −70 F and +200 F. Testing methods and apparatus are fully described and tables and graphs present results obtained. Moreover, a detailed discussion is included. R. G. Boiten, Holland

2671. **Louis Berger and John Gnaedinger, Thixotropic strength regain of clays,** Amer. Soc. test. Mat. Bull. 160, 64–68 (Sept. 1949).

Undisturbed samples of Grand Forks, N. D. clays were paraffined to preserve their natural state until tested. Unconfined compression strength tests were made after various remolding periods. All specimens showed increased brittleness and a decrease in per cent axial strain at maximum strength with increased storage time. Effect of moisture content, as would be expected, was significant. Tabulated properties of clays tested, as well as curves relating variables investigated, are included and fully discussed. J. M. DallaValle, USA

2672. **Robert F. Blanks, The use of Portland-pozzuolan cement by the bureau of reclamation,** J. Amer. Concr. Inst. 21, no. 2, 89–108(Oct. 1949).

Tests with Portland-pozzuolan cements show less heat generation and higher tensile strength and extensibility than those with normal Portland cements. Furthermore, substitution of 20 per cent of the cement by certain types of pozzuolans reduces expansion due to alkali-aggregate reactions by more than 75 per cent. All these properties are advantageous in production of mass concrete. P. P. Bijlaard, USA

2673. **J. R. Leslie and W. J. Cheesman, An ultrasonic method studying deterioration and cracking in concrete structures,** J. Amer. Concr. Inst. 21, no. 1, 17–36 (Sept. 1949).

Author describes an apparatus called the soniscope which was designed especially for nondestructive testing of concrete structures. It operates on a pulse transmission technique with an operating frequency of 20 kc/sec. Vibrations are generated by shock exciting a rochelle salt transducer having a rubber window. Contact is made with test piece by wetting its face with water or oil. The generator is physically small so that little directivity is obtained in the transmittal wave packets. Compressional, shear, and Rayleigh waves are generated simultaneously. Receiver is like generator. Basic measurement made is transit time of wave packets from generator to receiver. An accuracy of 3 per cent is achieved, with penetrations up to 50 ft in good concrete. Observed times of transits can be used to determine location and extent of cracks and elastic constants of specimen. It is concluded that there is a close correlation between modulus of concrete and its quality, a good block always showing a high modulus. Horace M. Trent, USA

2674. **I. T. Koudriashoff, Manufacture of reinforced foam concrete roof slabs,** J. Amer. Concr. Inst. 21, no. 1, 37–48 (Sept. 1949).

Russian type of lightweight concrete described uses a rosin-glue emulsion to preserve air voids before the initial set of the cement. Shrinkage is decreased and strength increased through high-pressure steam curing. Autoclave-treated foam concrete used in production of precast industrial roof slabs has a unit weight of 47 lb per cu ft and a compressive strength of over 500 psi. The lightweight slabs, used in a load-carrying capacity and as insulation, reduced construction time by 50 per cent and costs by as much as 20 per cent. Test data on roof slabs and production procedures are also described. From author's summary

2675. **Louis Chassevent, Study of plaster volume variation during setting and after** (in French), Rev. Mater. Constr. no. 405, 188–194; no. 406, 219–224; no. 407, 267–272; no. 408, 304–308 (June, July, Aug., Sept. 1949).

Author presents results of an extensive investigation of expansion and shrinkage of gypsum plasters and factors which affect these changes in volume. It was found that for any given plaster, expansion during hardening was greater when cured under water and the initial shrinkage small. A larger shrinkage during setting but a smaller expansion during hydration was exhibited by samples into which air was free to penetrate from start of setting action. These two extremes of curing condition provide limits to the expansion that may be expected for a given set of conditions. An increase in ratio of plaster to water (mix) and presence of unbaked gypsum and argillaceous materials were found to increase the facial expansion. Author describes in detail all experimental procedures used. Frank J. Mehringer, USA

Mechanics of Forming and Cutting

2676. **F. E. Stokeld, The hot forging and hot stamping of aluminum and its alloys,** J. Inst. Metals 76, part 5, 453–472 (1950).

General review of aspects of forging aluminum alloys. G. Gerard, USA

2677. **H. W. Swift, Stresses and strains in tube-drawing,** Phil. Mag. 40, no. 308, 883–902 (Sept. 1949).

Author considers sinking of tubes pushed or pulled through a die of constant angle. Friction and a special form of strain hardening are taken into account. Comparison is made with a few experiments and with simpler theories which ignore changes in tube-wall thickness. However, wall of tube is assumed thin so that bending stresses are neglected and pressure exerted by die is small compared with hoop stress. Maximum shearing stress is taken as criterion of yield and further plastic deformation after it is shown that when thickness changes, friction, and hardening are ignored, results differ only slightly from those obtained with the Mises criterion. The conclusions are that, except for tubes pushed through a die to reductions above 20 per cent, error involved in neglecting thickness changes in stress calculations is remarkably small. D. C. Drucker, USA

2678. William Schroeder and D. A. Webster, Press-forging thin sections: Effect of friction, area, and thickness on pressures required, J. appl. Mech. 16, 289-294 (Sept. 1949).

Measurements are reported of pressures needed to compress circular blanks having diameter thickness ratios extending up to 100. Tests cover a variety of lubricants at room and elevated temperatures. Factor by which observed pressure exceeds current yield stress is compared with that given by Siebel's analysis, suitably modified when frictional stress reaches its limiting value of shear-yield stress of blank material. Reasonable coefficients of friction are deduced. R. Hill, England

Hydraulics; Cavitation; Transport

(See also Revs. 2697, 2701, 2704)

2679. Richard C. Tolman, The effect of droplet size on surface tension, J. chem. Phys. 17, 333-337 (Mar. 1949).

Variation of surface tension with droplet radius is studied starting from Gibbs relation between surface tension, thermodynamic potential and superficial density. Superficial density Γ is defined as difference—per unit area of dividing surface—between actual mass of fluid present and mass which would be present if the two phases had constant density up to Gibbs surface of tension as dividing surface. Previous work by author indicated that Γ is positive. A new surface at a distance δ from surface of tension is defined so that if this surface is taken as the dividing surface, then Γ vanishes. From previously obtained values of Γ, δ is shown to be of the order of intermolecular distances. Author then assumes that δ will be independent of droplet radius and shows from this assumption, his previous estimates of the sign of Γ and Gibbs relation that surface tension should decrease with decrease in droplet radius. Arthur Kantrowitz, USA

2680. O. Kirschmer, Loss of pressure in high-pressure pipes and in open channels (in French), Rev. Gén. Hydr. 15, no. 51, 115-138 (1949).

This is an extensive review of the state of knowledge of frictional resistance in channels with and without a free surface. For smooth and rough closed channels, author presents Nikuradse data and the friction formulas of Prandtl and von Kármán based on these data. He concludes that the problem may be regarded as solved. In case of open channel, reference is made principally to the recent systematic work (1945) of Varwick who measured friction coefficients for flow in an open trapezoidal channel for various values of Reynolds number, hydraulic radius, and artificial roughness. It was found that, for smooth channels, free surface has little effect on the friction coefficient. With roughness, however, friction coefficient is considerably greater for open channel by an amount depending on magnitude of roughness, independent of Reynolds number. This remarkable result is explained in terms of secondary flows in the open channel.

Reviewer believes that paper contains several faults. One concerns the treatment of dimensional analysis, on basis of which author concludes that friction coefficient is expressible in form of a product of powers of dimensionless parameters upon which it depends. Actually, one can only conclude from a dimensional analysis that friction coefficient is a function of these dimensionless parameters. Another concerns author's criticism of the logarithmic law for turbulent friction, on basis that it fails to yield zero velocity at wall and predicts a zero value for friction coefficient for infinite Reynolds number, contrary to limiting value for friction coefficient of 0.012 proposed by certain authors. The answer to first objection is that logarithmic velocity-distribution law is applicable only to turbulent part of boundary layer and should not be applied at wall, in neighborhood of which flow is laminar. Second objection is one principally due to Telfer, but reviewer believes that there is no foundation for accepting the nonzero asymptotic value of friction coefficient [see discussion of Frictional resistance and ship resistance similarity, by E. V. Telfer, Trans. Instn. naval Arch. (1949)]. Louis Landweber, USA

2681. S. Irmay, On steady flow formulae in pipes and channels, Int. Assn. for hyd. Str. Res. III-3, 11 pp. (Sept. 1949).

An attempt is made to obtain simpler formulas to compute friction losses in pipe lines and open channels.

A new coefficient $c = 1/2(f)^{1/2}$ (f = friction factor) is introduced. A general solution is found for steady turbulent flow, including smooth, transition, and rough pipe flow. Roughness criterion is maximum and not average roughness height.

André L. Jorissen, USA

2682. H. L. Uppal, A study of certain aspects of transportation of bed load in lined canals, Int. Assn. for hyd. Str. Res. II-4, 25 pp. (Sept. 1949).

A summary of data obtained from study of bed load movement in canals and in model channels of various shapes and surfaces. Of particular interest are data taken on single marked particles of various shapes and sizes whose progress was followed through channels. Above data plus additional data on bed load movement in pressurized tunnels are presented in tabular and graphical form accompanied by photographs. No analytical treatment is given. Further work now in progress is outlined.

Dwight F. Gunder, USA

2683. C. Inglis, The effect of variations of charge and grade on the slopes and shapes of channels, Int. Assn. for hyd. Str. Res. II-1, 10 pp. (Sept. 1949).

The empirical Inglis-Lacey formulas for design of irrigation canals in alluvium (carrying suspended load) are mentioned. On basis of experiments and experiences from India, author discusses effect of seasonal variations in discharge, grade, and charge (load/discharge) on canals said to be "in regime," and effect of canal headworks on regime of rivers.

Hans Thygesen Kristensen, Denmark

2684. Cl. Carry, Average velocity of materials in suspension (in French), Int. Assn. for hyd. Str. Res. II-8, 15 pp. (Sept. 1949).

At Stockholm meeting of IAHSR in 1948, author's superior at Neyrpic laboratories at Grenoble, Pierre Danel, discussed fact that mean velocity of suspended-sediment transport is invariably smaller than that of the flow itself. Author presents in detail the analytical basis of this circumstance for flow in both open channels and closed conduits. In each instance, the rather complex relationships are reproduced graphically in the form of ratio of mean sediment velocity to mean flow velocity vs. ratio of fall velocity to shear velocity for various values of the Chezy discharge coefficient. While all curves approach sediment-velocity ratio of unity as a limit for small values of fall-velocity ratio, they tend to spread widely as latter ratio becomes large, varying between limits of zero and unity as the Chezy coefficient (proportional to ratio of mean flow velocity to shear velocity) increases.

Hunter Rouse, USA

2685. A. Craya, Schemes of suspension in variable regime (in French), Int. Assn. for hyd. Str. Res. II-7, 19 pp. (Sept. 1949).

Most analytical treatments of suspension of dispersed particles by fluid turbulence are restricted to conditions of uniform steady flow. Author extends analysis to particular cases of uniform unsteady flow for which specific solutions can be obtained. It is assumed throughout that fluid is semi-infinite in extent (i.e.,

bounded by either a free surface or a horizontal bed); that at initial instant fluid is either clear or in an equilibrium state of suspension; that mechanism of suspension is fully described by the customary diffusion function; and that diffusion coefficient is constant with time and space. After a brief comparison between theory of Brownian movement and that of turbulent diffusion, author derives relationships of point concentration against time for following cases: (1) Development of an equilibrium state of suspension by entrainment from bed; (2) cessation of suspension (decantation) through deposition upon a bed; (3) diffusion resulting from introduction of material at free surface; and (4) transition from one equilibrium state of suspension to another. As author is careful to point out, the simplified relationships which result should be applied with caution to actual circumstances which depart appreciably from those hypothesized. At the same time, it is anticipated that the functions will be practically useful in estimating times required for establishment of various equilibrium conditions.

Hunter Rouse, USA

2686. L. Escande, **The influence of the connection between surge tank and head race on the pressure conditions in a hydroelectric plant. Effect of the throat** (in French), Houille blanche 4, 457-478 (July-Aug. 1949).

By means of known Bergeron graphical method, excess pressures at toe of a surge tank are evaluated for different numerical situations and for particularly onerous closures. Results obtained show that, for closures occurring in practice, a throat at toe of surge tank does not lead to troublesome increases of pressure in gallery. An approximate formula is given for excess pressure. Losses of head in throat produce considerable increases of pressure, if closure is rapid and initial discharge is great (a case very improbable in practice). Great excess pressures in gallery are, on the contrary, conditioned in any case by presence of a long extent of conduit between gallery and surge tank.

Duilio Citrini, Italy

2687. K. K. Shalnev, **On the hydrodynamic pressure on a rotating blade in connection with cavitation calculation** (in Russian), Doklady Akad Nauk SSSR 67, 439-442 (July 1949).

A thorough analysis of forces experienced by a fluid particle streaming along a rotating blade leads to the conclusion that pressure distribution over such a wing differs from corresponding pressure distribution for a fixed wing. This difference is mainly due to a centrifugal force $dm\omega^2R$ (ω is angular velocity) and depends upon pressure coefficient $p' = 1 - W^2/W_1^2$, where W is the local velocity, W_1 velocity of entrance of liquid. For high negative values of p', neglect of centrifugal effect leads to an underestimation of slit cavitation. Besides, under this condition, development of cavitation on full-size machine and on model are not identical.

Georg P. Weinblum, USA

2688. H. Thygesen Kristensen, **Storm-water flow, non-permanent flow in sewers (the retardation problem)**, Int. Assn. for Hyd. Str. Res. I-7, 12 pp. (Sept. 1949).

Author points out that usual methods, e.g., the rational method, of calculating design flow for storm sewers are based on erroneous assumption that flow in sewer is steady. Actually, the flow is unsteady and waves or surges exist in system which tend to reduce actual times of concentration with the result that design capacities calculated by usual methods are too small and therefore on unsafe side. Author outlines a method for calculating capacities under conditions of unsteady flow but indicates that it is lengthy and suggests that before this type of analysis can be used in practice a simple approximate method of calculation must be developed.

Vito A. Vanoni, USA

2689. L. J. Tison, **Origin of ripple-marks and of sand-banks under the action of currents** (in French), Int. Assn. for Hyd. Str. Res. 11-13, 15 pp. (Sept. 1949).

Author first shows importance of velocity fluctuations in forming sand ripples. A particular feature of article is a description of so-called "primary" ripples (ripples produced in zone of erosion downstream from a bridge pier, river bend, etc.). Finally, a table is given summarizing numerous experimental results relating to ripple-mark dimensions which have been obtained by author.

A. Craya, France

2690. P. Bosz, **Hydraulic model test technique I, II** (in German), Arch. tech. Messen, no. 152, p. 25 (1947); no. 153, 39-40 (1948).

2691. James Robinson, **Variable speed hydraulic power transmission**, Proc. nat. Conf. indust. Hyd., 86-97 (Oct. 1947).

Incompressible Flow: Laminar; Viscous

(See also Revs. 2571, 2572, 2680, 2717, 2745, 2754, 2762, 2764)

2692. Garrett Birkhoff, **Hydrodynamics: a study in logic, fact and similitude**, Princeton University Press for University of Cincinnati, xiv + 186 pp., 1950. $3.50.

This unconventional book is concerned with some elements of technique of hydrodynamic research. Part of this technique is correct selection of relevant physical ingredients of the real phenomenon ("making good assumptions"). Incorrect assumptions lead to a "paradox" if they either do not determine one and only one solution, or if they provide a unique solution appreciably different from that observed. Chapters 1 and 2 (74 pp.) list and analyze a score or so of paradoxes, some classical (e.g., D'Alembert's), some pinned down in print for the first time. Their constructive contribution is to provide ideas when a change in assumptions looms necessary, and even to state some "rules" ("small compressibility can probably be approximated by its zero value, but small viscosity cannot; symmetric data may call for unsymmetric solutions").

Another element of the technique is good guessing at solutions of equations derived from assumptions. Principle of symmetry ("transformations leaving invariant the data leave invariant the solution") opens the door for group theory. Chapter III (39 pp.) deals with simplest and best known case of linear homogeneous transformations, i.e., with dimensional analysis. Author's treatment of Pi theorem will even attract the expert by its clarity and conciseness, as will a survey of modeling theory.

Chapter IV (35 pp.) exemplifies how guessing could be transformed into a systematic procedure; classical and very modern problems (as "automodel" solutions) are discussed.

Last chapter (30 pp.) reviews theory of the virtual mass concept. Its organic connection with body of book is not clear, and some group-theoretic considerations here would hardly interest the nonmathematical reader. Volume and range of references documenting this little book cannot remain unmentioned.

A. W. Wundheiler, USA

2693. Jerzy Litwiniszyn, **Stationary flows in heterogeneously unisotropic mediums**, Ann. Soc. Polo. Math. 22, 185-194 (1950).

Guided by analogy to theories of diffusion of heat or electric current, author sets up following equation for flow of an incompressible fluid in a porous medium: $V^i = A^{ij}p_{,i}$, $V^i{}_{,j} = 0$, or equivalently, $(A^{ij}p_{,i})_{,j} = 0$, where V^j is diffusion velocity, A^{ij} permeability tensor, and p pressure drop. He then obtains a Fourier-series solution for problem of flow in an isotropic semi-

infinite strip along whose finite edge pressure drop is linear, along whose infinite edges no flow takes place, and whose permeability falls off exponentially as a function of distance from finite edge.

C. A. Truesdell, USA

2694. Alexander Weinstein, Non-linear problems in the theory of fluid motion with free boundaries, Proc. Symp. appl. Math. 1, 1–18 (1949).

Paper reviews present knowledge of existence and uniqueness of solutions to problems of discontinuous flows with prescribed walls, i.e., jets issuing from prescribed boundaries and wakes behind obstacles placed in parallel flow. Attention is confined primarily to advances made since 1920. Two modes of approach are presented: (1) *Method of continuity* from which existence and uniqueness in the large have been proved for certain classes of symmetric jets, for wakes behind symmetric obstacles, and for wakes behind convex obstacles whether symmetric or not, provided obstacle is cut by a parallel to main stream in not more than one point. (2) A *variational method* due to Lavrentiev, apparently applicable only to symmetric wakes, but extended to include closed symmetric convex profiles, provided that obstacle is cut by every vertical in not more than one point in half the flow. Paper makes clear the small number of cases in which existence and uniqueness theorems have been proved. Thus, majority of problems of asymmetric wakes, wakes with re-entrant jets, and those with stagnation regions ahead of obstacle remain as yet unsolved.

Harold Wayland, USA

2695. Lydik S. Jacobsen, Impulsive hydrodynamics of fluid inside a cylindrical tank and of fluid surrounding a cylindrical pier, Bull. seism. Soc. Amer. 39, 189–204 (July 1949).

Paper contains thorough investigation of hydrodynamic forces developed on cylindrical tanks and piers during impulsive horizontal motions, such as might arise in an earthquake. Considered hydrodynamic problem is that of a circular cylinder partly filled with water and surrounded by a body of water of depth less than the height of cylinder. Base of cylinder is assumed to be subjected to impulsive but otherwise unspecified horizontal displacements.

In case of infinite depth, force is given by classical hydrodynamic theory. In case of limited depth, requirement that pressure be zero at free surface is found to modify this result, introducing a variation of horizontal pressure with depth. Solution of resulting potential-flow problem is obtained by a separation of variables in cylindrical polar coordinates. With vertical variation of pressure in form of a trigonometric series, it is found that radial variations for both internal and external flows are expressible by modified Bessel functions.

Following the theoretical development, a number of graphs are given showing numerical values of impulsive hydrodynamic masses, moments, and pressure distributions for various geometric proportions. Comparison of numerical results with experiments made on a model pier subjected to a small horizontal acceleration (12 per cent of gravity) shows satisfactory agreement. Paper concludes with a practical example showing an application of information provided by analysis to design of a concrete pier.

Robert T. Jones, USA

2696. W. Kaufmann, An energy calculation of the induced drag (in German), Ingen.-Arch. 17, 187–192 (1949).

In a previous paper [Sitzungsber. Bayer. Akad. Wiss. Mathem. Naturwiss. Abt., p. 109 (1946)], author calculated rolling up of a vortex sheet behind a wing of finite span into two spiral vortexes, replaced by vortex cylinders with circular cross section. Calculating the increment of kinetic energy of induced velocities behind wing per unit of time, induced resistance is found. At a considerable distance behind wing, the field is two-dimensional in a plane perpendicular to flight velocity. Kinetic energy is composed of two parts. One part, due to fluid outside vortex cores, is easily calculated by concentrating vorticity into center of cores. Another part, due to velocities inside cores, is calculated by an additional assumption about distribution of vorticity inside cores. For an elliptic lift distribution, results agree within 1 per cent with Prandtl's lifting-line theory.

R. Timman, Holland

2697. M. S. Plesset, The dynamics of cavitation bubbles, J. appl. Mech. 16, 277–282 (Sept. 1949).

Experiment has shown that flow about a torpedo-shaped body in a water tunnel can be adjusted to produce a few small cavitation bubbles just aft of nose portion. Elsewhere the water flow is continuous. This regime, which is of great interest in connection with the damage-producing mechanism of cavitation, has been studied both experimentally and theoretically. In setting up the equation of motion it has been assumed that each bubble behaves as if isolated in a spherically symmetrical pressure field of incompressible frictionless fluid. In order to integrate this equation, pressure at bubble wall has been assumed equal to vapor pressure at ambient temperature less pressure equivalent of surface tension; pressure at a distance from a bubble has been assumed to be unaffected by bubbles and has been computed from the value of pressure coefficient determined under noncavitating conditions; maximum bubble radius (where velocity is zero) has been obtained from photographic data. Theoretical values of bubble radius as a function of time are compared graphically with actual measurements. Good agreement is found, except near beginning and end of the bubble life. To discuss the interesting phenomena of collapse and rebirth of a bubble, theory must be extended to include compressibility effects.

Brandon G. Rightmire, USA

2698. Arthur W. Goldstein and Artur Mager, Attainable circulation of airfoils in cascade, Nat. adv. Comm. Aero. tech. Note 1941, 63 pp. (1949).

Considering airfoil cascades in two-dimensional incompressible flow, a method is developed for calculation of pitch-chord ratio and surface-velocity distribution which are consistent with the specification of profile thickness, maximum surface velocity, Reynolds number, inlet and outlet velocities and angles, suction surface diffusion rate, and trailing-edge loading. Analysis is based upon approximate boundary-layer theory, and both laminar and turbulent layers are considered. Numerical examples are given and compared with experiment.

W. G. Cornell, USA

2699. T. H. Havelock, The wave resistance of a cylinder started from rest, Quart. J. Mech. appl. Math. 2, 325–334 (Sept. 1949).

Expressions are obtained for wave resistance of a submerged circular cylinder moving normal to its axis when cylinder is suddenly started from rest and made to move with uniform velocity. This type of problem is of interest in ship-model tests, in which question arises how long it is before effect of starting conditions becomes inappreciable. Method of attack is based on an earlier paper by author [Proc. roy. Soc. A 93, p. 520 (1917)]. Expression for the resistance is obtained with Blasius theorem by integrating around a small contour near origin. It is noted that maximum steady resistance occurs at a speed for which wave length is 2π times depth of submergence. Results of calculations of resistance variation with time are presented for this speed and

for two lower speeds. A plot of results shows that resistance rises rapidly, initial resistance peak (or overshoot) is 1.11 times steady value for highest speed and about twice the steady value for one-quarter speed. Period of the resistance oscillation is four times wave length.

Author presents general approach to three-dimensional problem and, in case of a slender form, result is given in integral form. It is noted that for a considerable time after start, resistance reaches a known expression for steady resistance.

James M. Robertson, USA

2700. **Ya. B. Aron and Ya. I. Frenkel, Behavior of fluid drops (and bubbles) on the surface of a solid body. II. Experimental check of formulas for the critical angle of rolling and the minimum mass of drops (bubbles)** (in Russian), Zh. exp. teor. Fiz. 19, 807–813 (Sept. 1949).

In a preceding paper [same source, 18, 659–667 (1948); Rev 2, 891] Frenkel derived following relation for smallest angle at which a drop of a liquid will begin to roll down a solid surface: $2r\Delta\sigma = mg \sin \alpha^*$, where r is radius of circle where drop touches solid surface, m is its mass, g gravitational acceleration, α^* critical angle, and $\Delta\sigma$ the "adhesion" [$\Delta\sigma = \sigma$ (solid-gas) $+ \sigma$ (liquid-gas) $- \sigma$ (solid-liquid)]. In present paper, authors describe experiments to check this relation with mercury drops on glass, calcite, mica, talc, and sphalerite, waterdrops on calcite and mica, and air bubbles in water under aluminum plates. Gas in each case was air. For the drops, r, m and α^* were measured, for the air bubbles only r and α^*. Results indicate that for each size of droplet or bubble there exists a critical angle which increases as size decreases until a critical minimum size is reached, below which droplet or bubble is stationary for all angles. (This size corresponds to $2r\Delta\sigma = mg$.) In order to provide a complete experimental verification of above relation, it should presumably be shown that $mg \sin \alpha^*/2r$ is constant for each liquid-solid combination. This ratio is, in fact, used to compute $\Delta\sigma$ for the various combinations. However, given values are only averaged values with no measure for dispersion in data, although one statement would indicate that it was not considered large. Since the tables of measurements do not include r, it is not possible to find this from presented data.

J. V. Wehausen, USA

2701. **H. Fossett and L. E. Prosser, The application of free jets to the mixing of fluids in bulk,** J. & Proc. Inst. mech. Engrs. 160, no. 2, 224–232 (Sept. 1949).

A laboratory investigation carried out with scale models and confirmed by full scale tests has shown that a jet mounted near the bottom on the side of a cylindrical tank and directed across a diameter gives complete mixing of contents, if jet has an upward tilt to avoid stratification when mixing fluids of different densities. Required time T is approximately equal to $8D^2(QU)^{-1/2}$, where D is tank diam, Q and U discharge and velocity of jet at nozzle. Second component is introduced in tank through nozzle in part of T, while for remaining part of T, contents of tank are recirculated through it.

G. De Marchi, Italy

2702. **Georges Bouligand, On a case of a viscous liquid entrainment** (in French), C. R. Acad. Sci. Paris 226, no. 22, 1776–1778 (May 1948).

Continuing a previous investigation [same source, 226, 1571–1573 (1948); Rev 1, **1233**] author considers motion of a viscous incompressible fluid in an annular vessel of arbitrary cross section, initially at rest and then set into rigid rotation about its axis. He obtains an integrodifferential equation for the stream function of the motion relative to vessel. He indicates a proof that kinetic energy W of this motion is bounded by W_0e^{-Kt}, where K is a constant.

C. A. Truesdell, USA

2703. **G. I. Taylor, The mechanics of swirl atomisers,** Proc. seventh int. Congr. appl. Mech. 2, part I, 280–285 (1948).

Contribution discusses application of "perfect fluid" concept to flow through swirl atomizers. Author demonstrates from simple considerations that a perfect fluid theory is inapplicable. He concludes with the remark that his recent studies bear him out and indicate that "there is a strong axial flow along the core" of atomizer.

J. M. DallaValle, USA

2704. **G. Halbronn, Remarks on the theory of "Austausch" applied to the transport of materials in suspension** (in French), Int. Assn. for hyd. Str. Res. II-9, 6 pp. (Sept. 1949).

Considering the well-known general equation for suspended sediment in a turbulent stream,

$$\epsilon(dC/dy) = Cw,$$

where ϵ is exchange or diffusion coefficient, C mean concentration at elevation y above the bottom, w settling velocity of particles, author shows that this cannot satisfy equation of statistical equilibrium.

To overcome this contradiction, author assumes that mean transverse velocity is different from zero and develops a new equation:

$$\epsilon(dC/dy) = C(1 - C)w.$$

The application of both equations, assuming a logarithmic distribution of velocities, shows that concentration-distribution can be sensibly modified when percentage of suspended sediment becomes important.

G. A. T. Heyndrickx, Belgium

Compressible Flow, Gas Dynamics

(See also Revs. 2679, 2692, 2732, 2749, 2801, 2804)

2705. **S. V. Falkovich, Two-dimensional motion of a gas at large supersonic velocities,** Nat. adv. Comm. Aero. tech. Memo. 1239, 10 pp. (Oct. 1949).

Hodograph equation of a two-dimensional steady potential flow contains one variable coefficient if characteristic coordinates are used. Paper replaces this coefficient by the first term of a development which is valid for high Mach numbers and puts ratio of specific heats equal to 1.4. A Darboux equation, whose general solution can be given in a closed form, is then obtained. Furthermore, transonic and hypersonic similarity laws are derived by means of the hodograph.

Gottfried Guderley, USA

2706. **G. M. Roper, The yawed delta wing at incidence at supersonic speeds,** Quart. J. Mech. appl. Math. 2, part 3, 354–373 (Sept. 1949).

Solutions of linearized equations of supersonic flow about the flat yawed delta wing in lift are obtained, using a method of analysis similar to that developed by Stewart [Quart. appl. Math. 4, 246–254 (1946)] for symmetric delta with subsonic leading edges. Considered cases are: (1) Both leading edges supersonic, (2) both subsonic, and (3) one leading edge supersonic and one subsonic. This problem was first solved, in somewhat different fashion, by Hayes, Browne, and Lew [North Amer. Aviation Rept. NA-46-818 (June 26, 1947)].

William Pell, USA

2707. **S. I. Pai, Two-dimensional jet mixing of a compressible fluid,** J. aero. Sci. 16, 463–469 (Aug. 1949).

Equation of motion for laminar compressible flow in a two-dimensional jet, when linearized on assumption that velocity and temperature differ only slightly from those of surrounding stream,

leads to known equation of heat conduction and is readily solved. In the more general case, but with Prandtl number assumed unity, extension of von Mises' transformation, used by von Kármán and Tsien for problem of laminar boundary layer on a flat plate, is applied and solution is obtained by an iterative process of which first approximation is, conveniently, the solution of linearized equation. Results of a number of representative cases covering large and small temperature and velocity differences between jet and surrounding stream are presented. By carrying over concepts and assumptions of Reichardt's theory of free turbulence to compressible flow in a somewhat arbitrary fashion, it is shown that equations for a turbulent jet can be reduced to same form as those for a laminar jet, but they involve two constants which remain to be determined empirically.

A. D. Young, England

2708. E. V. Laitone, Hadamard's solution for an inclined body in linearized supersonic flow, J. aero. Sci. 16, 759–760 (Dec. 1949).

A solution for the velocity potential for linearized supersonic flow about an inclined slender body of revolution, which is analogous to subsonic doublet, can be obtained by using Hadamard's concept of "finite part of integral." This is pointed out again in present note where a form of this solution is presented; author quotes his previous work [J. aero. Sci. 14, 631–642 (1947)] for proof.

J. S. Isenberg, USA

2709. R. Timman, Linearization of the equations of two-dimensional subsonic compressible flow by means of complex characteristics, Proc. seventh int. Cong. appl. Mech. 4, 28–42 (1948).

The quasilinear differential equations of plane isentropic flow are considered. They are first put into characteristic form, and then linearized by transformation to the hodograph plane. Approximate solutions of the linear problem are discussed, and also Chaplygin's particular solutions in hypergeometric functions. A slight extension of a method used by Lin [Quart. appl. Math. 4, 291–297 (1946)] with the approximation $\gamma = -1$ (where γ has its usual meaning) is proposed.

W. R. Sears, USA

2710. G. N. Ward, Supersonic flow past thin wings. II. Flow-reversal theorems, Quart. J. Mech. appl. Math. 2, 374–384 (Sept. 1949).

Using total force equations derived in a previous paper [Rev 3, 1992], author proves two theorems on flow reversal. The first concerns effect upon drag and yaw forces of a thin symmetric wing at zero angle of attack when free-stream direction is reversed. The second pertains to lift of a thin wing of a less general plan form with same reversal of the free stream.

Hideo Yoshihara, USA

2711. H. S. Tsien and M. Finston, Interaction between parallel streams of subsonic and supersonic velocities, J. aero. Sci. 16, 515–528 (Sept. 1949).

In order to study problem of interaction between the compression shock and boundary layer, authors replace boundary layer with a uniform subsonic stream of finite width, bounded on one side by a solid wall and on other side by interface with a uniform supersonic stream of infinite extent. This simplified model is similar to one used by Howarth [Proc. Camb. phil. Soc. 44, 380–390 (1948)], but differs from it through fact that in Howarth's model the subsonic stream also was of semi-infinite extent. Fluid is assumed to be nonviscous and nonheat-conducting. Flow has been studied in two cases: (a) Incident compression wave from supersonic stream on interface; (b) wall is assumed to have a break, with a small constant slope after break. In both cases it is assumed the disturbance velocity potential is so small that linearization of the differential equations is possible. Disturbance velocities which originated in case (a) in supersonic stream and in case (b) in subsonic stream, are written in Fourier integral form, and therefore disturbance velocity potentials φ_1 and φ_2 can also be written in the form of a Fourier integral, either of the supersonic or of the subsonic stream. Assuming that, at the interface, the static pressure and the flow inclination are equal for both streams, authors determine the Fourier coefficients corresponding to the Fourier integrals which express φ_1 and φ_2. They further calculate the static pressures on interface and on solid wall. Authors develop a numerical application of method for various Mach numbers of streams.

In case (a) it appears that, except for local interaction, the incoming compression wave is reflected as if by a solid wall without the subsonic stream. Moreover, before the point of incidence, the flow is compressed, but after the point of incidence, the flow is expanded, and, therefore, in supersonic stream there is a series of compression wavelets sloping downsteam, ahead of incoming compression wave; after the incident wave, there is an expansion region. The pressure rises continuously along the wall, and most of pressure rise occurs before point of incidence of the compression wave. The flow configuration is, therefore, similar to what happens when a shock occurs over a laminar boundary layer, the case of the so-called λ shock. The region where there is strong interaction between supersonic and subsonic streams has a length directly proportional to width of the subsonic layer. According to authors, that is the reason why, if boundary layer is turbulent, interaction between the shock and a turbulent layer is different from that of the λ shock.

The numerical results for case (b) prove that while pressure on interface grows continuously, pressure distribution along the wall has a peak at starting point of the inclination; but most important result is the rather large compression ahead of the change in flow direction caused by the inclination. As phenomenon studied in (b) is approximately the same as one which occurs at the trailing edge of a supersonic airfoil, one can expect a pressure rise over the airfoil surface ahead of trailing edge above that predicted by a theory without taking into account effects of the boundary layer. Such propriety has been confirmed by Ferri's experiments.

Carlo Ferrari, Italy

2712. T. Y. Thomas, On conditions for steady plane flow with shock waves, J. Math. Phys. 28, 91–98 (July 1949).

A shock wave originating in plane uniform supersonic flow at vertex of a pointed obstacle and passing through vertex at all times, is fixed in plane of flow, and flow on downstream side of wave is steady if a certain relation exists between upsteam Mach number M and angle α which measures the inclination of shock line at vertex. Required relation exists if any one of a number of certain functions of the form $f_n(M, \alpha)$ is zero. A rather involved mathematical proof of this theorem is given.

C. W. Smith, USA

2713. Paul W. Huber, Cliff E. Fitton, Jr., and F. Delpino, Experimental investigation of moving pressure disturbances and shock waves and correlation with one-dimensional unsteady-flow theory, Nat. adv. Comm. Aero. tech. Note 1903, 1–65 (July 1949).

Pressure disturbances occur frequently in combustion and other related phenomena. While progress has been made in the development of the theory of unsteady flow in one dimension, little experimental verification has been attempted. This report contributes useful experimental information for comparison with theory, and shows that piezoelectric techniques can be success-

fully used with the shock tube. Schardin's method of calculating flow in shock tubes has been extended to include subsequent reflections and interactions of waves, so that a complete time history may be determined. Measurements were made in a shock tube by means of piezoelectric transient pressure recording equipment. Results showed good agreement between theory and experiment. It should be pointed out, however, that tests involved relatively weak shock waves.

Gordon N. Patterson, Canada

2714. **F. Schultz-Grunow, The Carnot shock loss in unsteady gas flow** (in German), Z. angew. Math. Mech. 29, 257–267 (Sept. 1949).

The Carnot shock loss (due to a sudden increase in cross section) is of fundamental importance for flow through valves. Author shows how to compute it, and conformity of his calculations with experiments of a periodically opened valve is presented. Other related phenomena are investigated; for example, reflection of a pressure wave at a finite sudden enlargement of cross section of a duct, or reflection of a pressure wave at an open end of a duct. By means of presented theory one may easily calculate the flow through a periodically opened valve and in duct attached to valve. Carnot losses at valve and at open end of duct, as well as friction losses, must be taken into account. Author assumes that stagnation pressures in container are not too high and that consequently velocities of propagation of all the shock waves are equal. Although this assumption simplifies the procedure, it constitutes a certain limitation upon applicability of results. Problem is not a stationary one, but author treats it as stationary by dividing the entire period into small parts, where each part is assumed to have a steady flow. Obtained results are then given in form of step curves. Comparison with performed tests shows good agreement. Method used is extremely simple in that it does not exceed elementary operations and uses only simplest equations from gas dynamics. Without any attempt to depreciate the results obtained by author, it is reviewer's impression that with use of some advanced mathematics, problem could be attacked as a fully nonstationary one and without any restrictions whatsoever concerning container pressures. Results presented in a form of continuous curves could then have a wider range of applicability.

M. Z. Krzywoblocki, USA

2715. **Henri Cabannes, Discontinuous potential flow of a perfect compressible fluid** (in French), C. R. Acad. Sci. Paris 229, no. 10, 510–511 (Sept. 1949).

Author asks under which conditions the flow downstream of a shock is still irrotational. Answer is given by a simple differential equation for shock direction. Result is illustrated by a simple example.

Gottfried Guderley, USA

2716. **M. J. Lighthill, The diffraction of blast. I**, Proc. roy. Soc. Lond. A 198, no. 1055, 454–470 (Sept. 1949).

A plane shock moves uniformly normal to a wall and encounters a corner where wall turns abruptly through a small angle δ. Author investigates behavior of the shock and of disturbed flow behind it after its encounter with the corner.

He proceeds by first linearizing the problem and then by transforming to a coordinate system in which corner and straight part of shock are fixed. In this system, equations are similar to those of Busemann's "conical flow" theory. In this system he determines boundary between disturbed and undisturbed flows behind shock and sets up boundary conditions. The flow equations themselves are transformed to Laplace's equation in what is essentially the pressure. After several further transformations a solution is obtained. Solution is then used to compute shock shape and pressure distribution for various shock Mach numbers. Results are given graphically. In particular, author shows that when the corner is convex, shock diffracts normally, varying only slightly from rectilinearity. In case of a convex corner, however, a shock "bridge" springs from the progressing shock wave back to wall in neighborhood of the corner, forming a configuration similar to the Mach of Y type of reflection.

D. P. Ling, USA

2717. **Edmund E. Callaghan and Dean T. Bowden, Investigation of flow coefficient of circular, square, and elliptical orifices at high pressure ratios**, Nat. adv. Comm. Aero. tech. Note 1947, 25 pp. (Sept. 1949).

In connection with heating or cooling of air streams by introducing high velocity gas or vapor jets perpendicularly to stream, the knowledge of the discharge coefficient of orifice from which jet is issuing is essential. Paper describes results of an experimental investigation carried out in a 2×20-in. duct, in which tunnel-air velocities of zero, 160, 275, and 380 fps were maintained and air jets were introduced from nine $^1/_{16}$-in. thick thin-plate orifices of circular, elliptical, and square shapes, between .375 and .625 in. in size. Investigated pressure ratios were 1.15 to 3.2, and jet total temperatures approximately 70 to 400 F. Relationships were established for the flow coefficients in terms of pressure ratio and jet Reynolds number. For pressure ratios up to choking, coefficients increased linearly with increasing pressure ratio; above choking, the straight line relation continued with a decreased slope. At constant pressure ratios, flow coefficient decreased with increasing jet Reynolds number. Ellipses yielded highest flow coefficients and no effect of stream velocity on flow coefficients was obtained.

Otmar E. Teichmann, USA

2718. **M. Z. Krzywoblocki, On steady, laminar two-dimensional jets in compressible viscous gases far behind the slit**, Quart. appl. Math. 7, 313–323 (Oct. 1949).

Problem defined by title, of a jet mixing with surrounding medium at rest, is solved by expressing velocity components, density, etc., in a series involving negative powers of x and a function $\eta \sim yx^{-1/2}$, and using iteration methods for determining successive approximations. Formulas are given up to third approximation. Reviewer believes that author's application of the over-all energy equation in a form which states that total flux of enthalpy across jet is alike at all cross sections, without accounting for the flux of kinetic energy, restricts solution to very low Mach numbers.

Ascher H. Shapiro, USA

2719. **R. Ladenburg, C. C. Van Voorhis, J. Winckler, Interferometric studies of faster than sound phenomena, Part II: Analysis of supersonic air jets**, Phys. Rev. 76, 662–677 (1949).

Following a brief summary and analysis of previous work, an investigation is made on axially symmetric and two-dimensional supersonic jets of air with free discharge. Shadowgraphs, interferograms, schlieren photographs, and measurements with impact and static probes provide the experimental data for analysis. Using interferometer techniques previously reported by authors, quantitative information is obtained on axially symmetric jets at pressure ratios up to 200:1 (absolute pressures in tank to receiver). Results at different pressure ratios are superimposed on a composite diagram which presents lines of constant pressure ratio (pressure at points in jet to tank pressure) on coordinates of Z/D (ratio of distance from orifice along the center line to orifice diam) versus R/D (ratio of radial distance from center line to orifice diam). All jets were closely the same in region bounded by orifice and an oblique line from orifice edge, an area of three-dimensional Prandtl-Meyer type of flow. At higher pressure ratios,

plane shocks and a three shock configuration develop. Measured strengths of shock waves agree with corresponding magnitudes calculated from Rankine-Hugoniot equations. Also, theoretical calculations for density ratios along axis of jet ($R/D = 0$) correspond satisfactorily with experimental values. A map of stream and Mach lines is constructed for one pressure ratio by a stepwise integration of equations of motion. Shape of jet is the result of interaction of pressure and inertial forces of outstreaming gas with receiver pressure force to produce successive overexpansion and overcontraction. Results from interferograms of a two-dimensional jet confined between parallel glass walls illustrate the limitations of this type of analysis. Boundary-layer and separation phenomena, if present, would hinder interpretation of interferograms due to unknown variation of air density along light path.

R. G. Folsom, USA

2720. **Gerald E. Nitzberg and Stewart Crandall, A study of flow changes associated with airfoil section drag rise at supercritical speeds,** Nat. adv. Comm. Aero. tech. Note no. 1813, 31 pp. (Feb. 1949).

A study is made of experimental pressure distributions and section characteristics for several moderately thick airfoil sections. A correlation appears to exist between the drag-divergence Mach number and the free-stream Mach number for which sonic velocity occurs at airfoil crest, the chordwise station at which airfoil surface is tangent to free-stream direction. It is found that, since the Mach number for which sonic velocity occurs at airfoil crest can be estimated satisfactorily by means of Prandtl-Glauert rule, a method is provided whereby drag-divergence Mach number of an airfoil section at a given angle of attack can be estimated from low-speed pressure distribution and airfoil profile. Method is used to predict with a reasonable degree of accuracy the drag-divergence Mach number of a considerable number of airfoil sections having diverse shapes and a wide range of thickness-chord ratios.

Pressure distributions and section-force characteristics of several moderately thick airfoil sections at Mach numbers above drag-divergence Mach number are analyzed. Some of characteristics of flow over these airfoils at supercritical Mach numbers are discussed.

From author's summary

2721. **Clinton E. Brown, The reversibility theorem for thin airfoils in subsonic and supersonic flow,** Nat. adv. Comm. Aero. tech. Note 1944, 9 pp. (1949).

In a simple and beautiful theory, author has improved an idea of M. Munk [Nav. Ord. Lab. Memo. 9624 (1948)] and obtained general reversed-flow theorems for lift and related quantities in a linearized fluid flow. Main principles used are those of superposition and of representation of drag in the wave and vortex systems, and theorems hold for very general bodies in either subsonic or supersonic flow. Kutta condition is essential to validity of the theorems.

The new theorems proved are, that for any body satisfying planar system or similar restrictions, the lift curve slope $dC_L/d\alpha$, the damping in roll C_{l_p}, and the damping in pitch c_{m_q} are unchanged by a reversal of the flow direction.

The theorem for lift is given briefly to exemplify the method. The drag of a wing D_1 and the drag in a reversed flow D_2 may be expressed as $D_1 = L_1\alpha - F_1$; $D_2 = L_2\alpha - F_2$, where L_1 and L_2 are the lifts, and F_1 and F_2 the subsonic leading edge thrusts. A suitable superposition of the perturbation solutions yields a system for which the drag is $D_1 - D_2 = F_2 - F_1$, superposition of the drags being permissible because of noninteraction of the wave or vortex systems, and superposition of leading edge thrusts being enabled by the Kutta condition. This gives $L_1\alpha = L_2\alpha$ which states the theorem.

Author points out an error in a paper of W. Hayes [Proc. sev. int. Cong. appl. Mech., London; North Amer. Av. Rept. AL-755], who had mistakenly concluded that this lift theorem could not be general. Other authors, for example G. N. Ward, A. Flax, and S. Harmon, have given similar theorems but with much less generality.

W. D. Hayes, USA

2722. **C. C. Lin, On the subsonic flow through circular and straight lattices of airfoils,** J. Math. Phys. 28, 117–130 (July 1949).

Author gives a general inverse method for constructing compressible subsonic flow over a straight lattice of airfoils of infinite extent, or a circular lattice of finite number of airfoils. Method is based upon linear pressure-volume relation and is an essential improvement of von Kármán-Tsien method by the author [Quart. appl. Math. 4, 291–297 (1946)] and Gelbart [Nat. adv. Comm. Aero. tech. Note 1170 (1947); Rev 1, **494**].

H. S. Tsien, USA

2723. **Gottfried Guderley and Hideo Yoshihara, The flow over a wedge profile at Mach number one,** AF tech. Rep. 5783, 37 pp. (July 1949).

Paper considers flow field about a wedge profile with free-stream Mach number unity at zero angle of attack. The simplified potential flow in hodograph plane is governed by known Tricomi differential equation of mixed type. Another gain of this simplification is existence of the transonic similarity law which was discovered by von Kármán in 1946. Similarity law yields three important results: (a) A meaningful generalization of aerodynamic behavior of vanishingly thin profiles is possible; (b) hodograph transformation from physical plane becomes exceedingly simple; and (c) a simplified boundary condition can be used.

Being linear in the hodograph plane, superposition of particular solutions can be applied, but unfortunately it leads to an infinite system of linear equations to be solved. The difficulty is overcome by introducing an equivalent boundary condition upon which an integral equation with singular kernel can be formulated. The integral equation is quite similar to one usually occurring in thin-airfoil theory and is solved by decomposing boundary conditions into Fourier series. Mathematical technique seems interesting.

Flow up to "limiting Mach wave" line is calculated with present method. Flow after limiting Mach wave, being supersonic, is constructed by method of characteristics. Rear portion of profile can be chosen arbitrarily within certain limits. Wave drag of such profiles is evaluated under optimistic assumption of potential flow. Of course, this is due to antisymmetric supersonic flow produced locally, even though the diamond profile is geometrically symmetrical fore and aft. As an example, the wave-drag coefficient C_D of a diamond profile of 10 per cent thickness is calculated as 0.0842 with optimum position of maximum thickness at 63 per cent of chord. Of course, the linearized theory would give infinite C_D at $M = 1$ for such two-dimensional flow.

Chieh-Chien Chang, USA

2724. **Eric Reissner, On the theory of oscillating airfoils of finite span in subsonic compressible flow,** Nat. adv. Comm. Aero. tech. Note 1953, 37 pp. (Sept. 1949).

Paper extends author's work on incompressible unsteady flow over wings of finite aspect ratio ["On the general theory of thin airfoil for non-uniform motion," NACA T.N. 946 (1944); "Effect of finite span on the airload distributions for oscillating wings—I," NACA T.N. 1194 (1947)] to the subsonic, compressible case. After carefully formulating the boundary conditions throughout plane of wing and at infinity (Sommerfeld "radiation" condition

and posing a harmonic time dependence, differential equation for velocity potential is reduced to the classical Helmholtz equation, and an appropriate solution is constructed with aid of free-space Green's function. This solution leads to the lifting surface integral equation for oscillating airfoil. While this equation has been given previously by H. G. Kussner ["Allgemeine Tragflächentheorie," Luftfahrtforsch. 17, 370–378 (1940)], present derivation exhibits motivation of the result from the boundary conditions with greater clarity and places it in a rather more useful form. This two-dimensional (i.e., two independent variables) integral equation is then transformed to a pair of one-dimensional integral equations, which may be regarded as being associated with problems of local flow at each spanwise station and with spanwise-circulation distribution, and are analogous, respectively, to Possio equation for two-dimensional, oscillating airfoil and to the Prandtl lifting-line equation. This reduction assumes a "sufficiently large" aspect ratio. (Reviewer's note: It appears, from the approximation, that the significant aspect ratio should be that of the rectangle which circumscribes the plan form, so that wings having appreciable sweep would be excluded.) While the solution of these equations is not treated in paper, it evidently will be a task of some magnitude. (Reviewer's note: Transformation introduced by author suffices to reduce the low frequency problem—i.e., neglecting terms of order $k^2 M^2 (1 - M^2)^{-2}$, where k and M are reduced frequency and Mach number, respectively—to an equivalent problem in incompressible flow, thereby furnishing an extension of the Prandtl-Glauert transformation. Used in connection with author's earlier result [op. cit.], this furnishes a powerful tool for examining the interaction of compressibility and aspect ratio corrections of aerodynamic forces due to unsteady flow. At higher frequencies, on the other hand, aspect ratio corrections are relatively less important.)

John W. Miles, USA

2725. **Isao Imai, Application of the w.k.b. method to the flow of a compressible fluid I,** J. Math. Phys. 28, 173–182 (Oct. 1949).

Equations of motion in hodograph plane for two-dimensional flow of a nonviscous compressible fluid, can be written in the form $\Psi_{tt} + K\,\Psi_{\theta\theta} = 0$; $K = (1 - q^2/c^2)(\rho_0/\rho)^2$, where Ψ is the stream function, q is velocity, and rest of symbols have their usual meanings. Author shows that with the transformations

$$\Psi = K^{-1/4}\psi \quad \text{and} \quad \tau = -\int K^{1/2} dt \qquad [1]$$

the equation becomes $\psi_{\tau\tau} + \psi_{\theta\theta} = k\psi$; $k = K^{-1/4}(d^2K^{1/4}/d\tau^2)$. For an adiabatic flow

$$k(q) = -\frac{1 - a^2}{4} q^4 \frac{8 - 2(1 + 3a^2) q^2 - (1 - a^2) q^4}{(1 - q^2)^3 (1 - a^2q^2)}$$

where $a^2 = (\gamma - 1)/(\gamma + 1)$, and this is $O(M^4)$ where M is Mach number of the undisturbed flow.

The advantage of transformation [1] is that for moderate speeds, near obstacles and at great distances from obstacle, either k is small or $k\psi$ can be neglected. Accordingly, approximate solutions can be obtained by neglecting the term $k\psi$ in transformed equation. Author discusses various problems in which method can be used advantageously for obtaining approximate solutions.

S. Chandrasekhar, USA

2726. **Henri Cabannes, On compressible flow in the neighborhood of sonic speed** (in French), C. R. Acad. Sci. Paris 229, 102–104 (July 1949).

It is known that the stream function which represents two-dimensional flow through a convergent-divergent nozzle is a multivalued function of the hodograph variables. Accordingly, the hodograph equation for stream function is reduced to a Tricomi equation—which will yield the desired type of solution—by assuming the compressibility law [1] ... $A(u) = (\gamma + 1)u$, where $u = {}_1\int^q \rho/q\, dq$, $A(u) = -1/\rho^2(1 - q^2/c^2)$ and q, ρ, c, γ are velocity, density, velocity of sound, and ratio of specific heats, respectively. Expressions for pressure and for density which follow from law [1] are given in terms of Bessel functions of one-third order and argument u. Law [1] is a good approximation to adiabatic law in the neighborhood of sonic speed. [See also similar work in Quart. appl. Math. 7, 381–397 (1950).]

J. S. Isenberg, USA

2727. **Pietro Teofilato, Extension of an hydro gas dynamic similitude to the flow with axial symmetry,** Proc. seventh int. Congr. appl. Mech., 19–27 (1948).

2728. **Donald D. Baals, Norman F. Smith, and John B. Wright, The development and application of high-critical-speed nose inlets,** Nat. adv. Comm. Aero. Rep. 920, 58 pp. (1948).

An investigation was undertaken in the Langley 8-ft high-speed tunnel to establish effects of nose-inlet proportions on critical Mach number and to develop a rational method for design of high-critical-speed nose inlets to meet desired requirements. The nondimensional ordinates of the B-nose inlet, which were developed in a previous investigation [J. V. Becker, NACA ACR (Nov. 1940)] to be optimum from standpoint of critical speed, were extended and modified slightly to improve the fairing. These ordinates, now designated the NACA 1-series, were then applied to a group of nose inlets involving a systematic variation of proportions.

Pressure distributions and critical Mach number characteristics are presented for each of tested nose inlets. Results of these tests show that length ratio (ratio of length to maximum diameter) of nose inlet is the primary factor governing maximum critical speed. Effect of inlet-diameter ratio (ratio of inlet diameter to maximum diameter) on critical speed is, in general, secondary; but this ratio has an important function in governing the extent of inlet-velocity-ratio range for maximum critical speed. Highest critical Mach number attained for any of the nose inlets tested was 0.89. Data are arranged in form of design charts from which NACA 1-series nose-inlet proportions can be selected for given values of critical Mach number and airflow quantity.

Selection charts and NACA 1-series ordinates are shown to be applicable to design of cowlings with spinners and to design of high-critical-speed fuselage scoops. Possibility of application of NACA 1-series ordinates to experimental development of wing inlets is also indicated.

From author's summary by J. V. Foa, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 2698, 2707, 2711, 2766, 2826, 2830)

2729. **J. Rotta, On the theory of the turbulent boundary layer** (in German), Mitt. Max-Planck-Institut Strömungsforschung, no. 1, 54 pp. (1950).

Paper gives procedure for computation of turbulent skin friction in boundary-layer flow with pressure gradient. Boundary layer is divided into inner and outer regions. Inner flow depends on friction coefficient, viscosity, and roughness following Prandtl's treatment. Outer flow depends only on friction coefficient and pressure gradient. For outer flow, author studies similar solutions found when velocity of free stream varies as mth power of x, and ratio of shear velocity to free-stream velocity is constant. Boundary-layer thickness is proportional to x and

definite relationship required between Reynolds number, wall roughness, and other parameters. In general case, these solutions are used as approximations over short intervals of x. Separation occurs when $m = -0.2$ as compared with $m = -0.091$ for laminar layer. Although this theory indicates an effect of shear velocity ratio, experimental data show small effect compared with that of pressure gradient when suitable nondimensional parameters are used. Paper also discusses turbulent energy, dissipation, momentum, and energy relations. Treatment is good summary of approach used by Göttingen group.

Hugh L. Dryden, USA

2730. **H. Ludweig, Instrument for measuring the wall shearing stress of turbulent boundary layers** (in German), Ingen.-Arch. 17, 207–218 (1949). [Eng. transl.: Nat. adv. Comm. Aero. tech. Memo. 1284 (1950).]

Author starts with a review of older experiments for measuring shearing stress at wall, and then shows that, for turbulent boundary layers with pressure gradients, only the part of velocity profile next to wall depends on shearing stress at wall. Therefore, the heat transfer of a small heated element whose temperature profile extends only over this next-to-wall part of velocity profile is a measure of wall shearing stress. New apparatus is constructed on this basis. An approximate investigation, assuming temperature profile wholly within laminar sublayer of turbulent boundary layer, yields the relation: $\alpha \sim \tau_0^{1/3}$, i.e., coefficient of heat transfer is proportional to third root of wall shearing stress. This relation agrees with experiment. Because temperature profile is practically somewhat thicker than laminar sublayer, a calibration of apparatus is necessary.

Work contains a detailed description ot apparatus, consisting essentially of a well-insulated piece of copper in surtace of wall. In this piece of copper are inserted a thermocouple and an electrically heated wire, so that it is possible to measure temperature of copper piece and rate of heat loss. These measurements determine amount of wall shearing stress. If the copper element has a slender rectangular form, direction of the wall shearing stress can be determined by turning the element. Fritz W. Riegels, USA

2731. **Ryosuke Hama, The energy distribution in the spectrum of turbulence** (in Japanese), Rep. Inst. Sci. Technol. Tokyo 3, 223–228 (July-Aug. 1949).

Author calculates the spectral distribution of energy in turbulence by use of a model suggested by theory of heat radiation. Equations of hydrodynamics are not taken into account, but following hypotheses are introduced: (1) Only energy amounts $m\epsilon$ ($m = 0, 1, 2, \ldots$) are possible; (2) distribution of energy obeys Maxwell-Boltzmann law. Then, assuming that the energy element ϵ is proportional to some power of wave length, and using dimensional considerations, author arrives at a formula for spectral distribution which vanishes at zero frequency (i.e. at infinite wave length) and coincides fairly well with existing experimental values. Itiro Tani, Japan

2732. **Dean R. Chapman and Morris W. Rubesin, Temperature and velocity profiles in the compressible laminar boundary layer with arbitrary distribution of surface temperature,** J. aero. Sci., 16, 9, 547–565 (Sept. 1994).

Paper represents an impressive effort, not only to devise a method for predicting behavior of laminar boundary layers at high speed with variable surface temperature, but also to use resulting method in exploring phenomena that are present in such cases. Attention is confined to flows without pressure gradient. Analysis rests on usual assumptions of compressible boundary-layer theory. Viscosity is taken to be proportional to absolute temperature. The constant of proportionality can be fitted to either free stream or wall conditions, with latter being recommended. The coordinate normal to boundary layer is replaced by stream function as independent variable, and under these circumstances the conditions for velocity distribution are independent of temperature distribution. In fact, the solution for velocity distribution is obtained in terms of the Blasius solution for incompressible flow. Result is inserted in equation for temperature and a solution obtained in terms of coefficients of a power series for wall temperature distribution. Explicit methods are given for returning to the original independent coordinate system and for finding heat transfer, skin friction, velocity, and temperature profiles, etc.

A specific example is investigated for case of a plate with leading edge heated well above equilibrium temperature and with wall temperature decreasing to a value well under equilibrium at trailing edge. Free-stream Mach numbers of 0.5 and 3.0 are considered. It is found that variable surface temperature has a large effect. Both local and over-all heat-transfer coefficients are quite different from those calculated, neglecting what has taken place upstream in boundary layer. (Over a surprisingly large portion of plate, the heat transfer is in opposite direction to that calculated, using a temperature difference from equilibrium as a base.) It is pointed out that a heat-transfer coefficient based on a temperature difference from equilibrium is inappropriate for flows with variable surface temperature.

It is interesting to note that a reasonable estimate of heat transfer on a surface with variable temperature can be made by assuming that heat transferred is retained in boundary layer, and that effective temperature for local transfer is thus raised by the corresponding increase in mean boundary-layer temperature. In this way, results from analyses for uniform wall temperatures can be used with improved accuracy. Francis H. Clauser, USA

2733. **August Wilhelm Quick and K. Schroder, Behavior of the laminar boundary layer for periodically oscillating pressure variation,** Nat. adv. Comm. Aero. tech. Memo. 1228, 9 pp. (Sept. 1949).

Experimental results suggest that, e.g., slight fluctuations in free-stream velocities, as they occur in wind tunnels, or slight wavelike deviations of outer wing contours may exert strong effects on the extent of laminar boundary layer on the body and thus on drag. Application of Pohlhausen method to this problem does not seem to give reliable results. Authors use another method, developed formerly by second author, reference to which unfortunately is not given in paper. This reports on results of calculation of a special example: flow about a plate where undulation starts only after an initial plane section. Results are shown in drastic graphic representations of variations of boundary-layer profiles, the streamlines in boundary layer including small vortex centers and reverse flow, the displacement and momentum thickness, the shearing stresses and the retroaction of the undulation of the displacement thickness on pressure distribution. This retroaction must be taken into consideration.

Calculations prove that the laminar boundary layer is actually extremely sensitive to slight variations in pressure distribution and tends easily toward separation. Transition of the boundary layer from laminar to turbulent is caused by onset of reversal flow followed by a vortex formation which, in turn, may be produced by fluctuation of free stream and by wall roughness. If a monotonously increasing pressure rise exists, the point of transition, caused by additional pressure oscillation, will lie generally ahead of the separation point of the laminar boundary layer which results from calculation with undisturbed pressure distribution.

W. Spannhake, USA

2734. **Ratip Berker, On a solution of the equations of the boundary layer** (in French), Proc. seventh int. Congr. appl. Mech. 2, part I, 83–96 (1948).

Nearly all known exact solutions of Prandtl's boundary-layer equations are "similar" solutions for which the profile of the velocity depends only on one parameter $\eta = y/f(x)$ (x, y Cartesian local coordinates). In this case boundary-layer equations can be reduced to ordinary differential equations. In 1908 Blasius treated the flat plate in a tangential flow as the first exact example with $f(x) = (x)^{1/2}$, plate being assumed infinite in $+x$-direction. With a somewhat modified assumption, author solves problem of finite plate by means of elliptic functions, as used formerly by G. Hamel, to solve a related differential equation (laminar flow between two straight nonparallel walls). With respect to mathematical details, especially how author satisfies conditions at limit of boundary layer, paper must be studied, since it is purely formal and contains no diagrams. Manfred Schäfer, Germany

2735. **R. Ladenburg and D. Bershader, On laminar and turbulent boundary layer in supersonic flow,** Rev. mod. Phys. 21, 510–515 (July 1949).

Paper reports results of experiments, using an interferometer, to determine characteristics of boundary layer on a flat plate at a free-stream Mach number of 2.3. Usual assumptions are made in conversion of density to velocity profiles in both laminar and turbulent boundary layers.

Results show that, in turbulent boundary layer at a local Reynolds number of 3×10^6, velocity distribution follows the $1/9$th power law. In case of laminar boundary layer, measurements are shown to lie close to Crocco's theoretical curve, except at distances close to plate (reviewer has noted that if correct relation between viscosity and temperature is used in determination of theoretical curve, experimental points lie closer to it than is apparent from Fig. 11 in paper). It is also shown that for local Reynolds numbers greater than 2×10^6, boundary layer was turbulent, while for Reynolds numbers below 6×10^5, boundary layer was laminar, indicating that in this case a large region of transition was present. G. M. Lilley, England

2736. **Hiroshi Tsuji, On the solution of the laminar boundary layer equations** (in Japanese), Rep. Inst. Sci. Technol. Tokyo 3, 239–241 (July-Aug. 1949).

It is shown that the solution of laminar boundary-layer equations may be reduced to that of ordinary differential equations, provided that xdM/dx is expressed as a polynomial of $M = x(U)(dU/dx)$, where x is the distance measured along the wall, and U is the velocity outside the boundary layer. This condition is satisfied by all solutions already obtained: (1) $U = x^m$ by Falkner and Skan [Rep. Memo. aero. Res. Counc. Lond. no. 1314 (1930)]; (2) $U = 1 + cx$ by Howarth [Proc. roy. Soc. Lond. Ser. A 164, 547–579 (1938)]; (3) $U = 1 + cx^n$, $n = 2, 4, 8$ by Tani [J. phys. Soc. Japan 4, 149–154 (1949)]; (4) $U = xe^{-x}$ by Falkner [Rep. Memo. aero. Res. Counc. Lond. no. 1884]. It is suggested that the case $U = x^n e^{-x}$ may also be solved along similar lines. Itiro Tani, Japan

2737. Dimitri **Riabouchinsky, The phenomenon of striation of the boundary layer and turbulence,** Proc. seventh int. Congr. appl. Mech. 2, part I, 97-111 (1948).

Author describes method published by him in 1909 for visualizing fluid motions in neighborhood of rigid boundaries with subsonic and supersonic regimes, and phenomenon of striation of the boundary layer. Representations show flow pattern of a flat plate normal to flow, longitudinal and transverse striation of boundary layer in a gaseous jet, and traces of adjoined eddies tangent to edge of disk uniformly revolving in their own plane. Last problem is dealt with in some detail. Simple empiric formulas are given for dependence of frictional resistance on angular velocity ω in the two cases of laminar and turbulent regimes of boundary layer; formulas are in good agreement with present experiments. Relation $Vr^{3/2} = 0.16\ \omega R^{3/2}$ (V mean velocity, R radius of disc, r local vector of the field) is deduced and proposed for discussion. Manfred Schäfer, Germany

2738. **A. Favre, Statistical measurements of time correlation** (in French), Proc. seventh int. Congr. appl. Mech. 2, part I, 44–55 (1948).

In the study of turbulence, author wishes to measure statistically the correlation in space and time between two independent, variable quantities which are observed in a local distance at variable times. The principle of his apparatus is that the voltages in two electrical hot-wire anemometers u_t and v_t are registered on separate magnetic ribbons moved with constant speed V. The records are taken up at variable distances L and l respectively by pickups, and after amplification, are applied to the horizontal and vertical plates of a cathode-ray tube where u_t and v_t appear with the variable time displacement $T = (L - l)/V$. The correlation coefficient $r = u_t \cdot v_{t+T}$ is measured. Apparatus is described in detail, and measured results are reproduced. The correlation coefficient in time and the turbulence spectrum of air flow in a blower are shown in a diagram.

Manfred Schäfer, Germany

2739. **Theodore von Kármán and C. C. Lin, On the concept of similarity in the theory of isotropic turbulence,** Rev. mod. Phys. 21, 516–519 (July 1949).

Authors analyze spectrum of an isotropic turbulence and its change during process of decay. By dimensional analysis the energy spectrum is expressed in the form $F(k) = v^2 l\ \varphi(kl)$, where k is wave number, v a typical velocity, and l a typical length. Problem is to connect latter two quantities with measurable ones, such as viscosity ν, rate of energy dissipation ϵ, and Loytsiansky's invariant J_0. According to experimental evidence, it is not possible to express v and l by unique relations for all k values and whole process of decay. Therefore, authors introduce three sets of characteristic velocities and lengths, which they call V^*, L^*; V, L and ϑ, η connected respectively with lowest, medium, and high frequency ranges. They assume the hypothesis $V^* L^* = VL = D$ and call parameter D the "eddy diffusion coefficient of the turbulence mechanism." They assume that for higher frequencies, only significant parameters are ν and ϵ, and for the lowest, J_0; thus they get:

$$\vartheta = (\nu\epsilon)^{1/4} \qquad \eta = (\nu^3/\epsilon)^{1/4}$$
$$V = (D\epsilon)^{1/4} \qquad L = (D^3/\epsilon)^{1/4}$$
$$V^* = (D^5/J_0)^{1/4} \qquad L^* = (J_0/D^2)^{1/4}$$

(the values ϑ, η correspond to Kolmogoroff's known laws). They suggest following description for the process of decay: (1) Early stage: turbulence field is created by some mechanism which produces individual eddies; early stage is first period of decay, after homogeneity and isotropy are established; in this stage η/L = const; (2) intermediate stage, in which L/L^* = const; (3) final stage, tending toward complete similarity at extremely low Reynolds numbers. J. Kampé de Fériet, France

2740. **Raymond Comolet, On laminar flow in a short tube** (in French), C. R. Acad. Sci. Paris 229, 342–343 (Aug. 1949).

Note treats an experimental study of effect of curvature of axis of a short tube on stability of laminar flow. Experiments demonstrate that, in absence of more serious disturbances, it is

the geometric characteristics of tube which determine value of critical Reynolds modulus. Warren A. Hall, USA

Aerodynamics of Flight: Wind Forces

(See also Revs. 2573, 2720, 2728, 2755, 2757)

2741. M. J. Turner, Aerodynamic theory of oscillating swept-back wings, J. Math. Physics 28, 280–293 (1950).

Extension of work of E. Reissner (Nat. adv. Comm. aero. Tech. Note 946 (1944); 1194 (1947); Rev 1, **718**) on oscillating thin finite-span wings in incompressible inviscid flow, to swept-back wings. Case treated is that of a uniformly sheared rectangular wing, i.e., one whose plan form is a parallelogram. In oblique coordinates analysis resembles Reissner's, but slightly generalized approximations are required. It is found that Reissner's principal results might be used with minor modifications, but certain new functions would have to be tabulated. The basic integral equation for a wing consisting of two sweptback trapezoidal panels is set up but is not treated in detail.

W. R. Sears, USA

2742. William T. Sawyer, Experimental investigation of a stationary cascade of aerodynamic profiles, Mitt. Inst. Aerodyn. Eidgenoss. Tech. H. Zurich no. 17, 78 pp. (1949).

This latest of a series of notable contributions of Institute for Aerodynamics of the ETH presents results of studies undertaken "to seek a more exact basis for evaluation of performance of a single cascade." A modified Ackeret continuum cascade theory is developed, distinguished from original theory by bell-shaped concentrations of vorticity and source fields arranged along profile median lines, instead of rectangular or trapezoidal concentrations. A numerical calculation of accelerating cascade to be tested, designed for given characteristics, is presented complete with time-saving auxiliary charts. Results are compared with Euler theory.

Principal feature of cascade test equipment is a force balance based upon Amsler principle of elimination of dry friction by relative motion of supporting surfaces, which permits measurement of force component in any direction with 0.1 per cent accuracy.

Cascade tests were made at various intensities and scales of turbulence at Reynolds numbers up to 23,000. Transition and separation were studied with kerosene and lampblack and with the stethoscope. Force measurements were compared with integrated static pressure distributions and momentum traverses. Drag was determined from balance measurements, integration of pressure distribution, and from momentum wake traverses. Good agreement between these is obtained upon elimination of end effects. An expression for profile efficiency was derived, based upon tangential and normal forces on cascade profiles.

J. R. Weske, USA

2743. H. H. B. M. Thomas and M. Lofts, Application of thin aerofoil theory to controls having set-back hinge balance, with an analysis of wind-tunnel data on aerofoils of finite thickness, Rep. Memo. aero. Res. Counc. Lond. no. 2256, 33 pp. (July 1945, publ. 1949).

Theory of thin airfoils, as developed by Birnbaum, Munk, Glauert and others, is applied to problem of determining characteristics of wing control surfaces having hinge axes set back of their leading edges in order to provide a balance of required hinge moment. Theory is developed on basis of a mean camber line broken at point of maximum thickness of control-surface nose profile.

Theoretical results are used as a guide for analysis of wind-tunnel test data, with inclusion of semiempirical corrections for effects of finite span of both wing and its control surface. Results indicate that theory provides a reasonably satisfactory basis for analysis of test data, although additional tests are needed to provide complete confirmation of this tentative conclusion.

M. J. Thompson, USA

2744. Stüper, Flight experiences and tests on two airplanes with suction slots, Nat. adv. Comm. Aero. tech. Memo. 1232, 104 pp. (Jan. 1950).

Measurements on two full-scale airplanes equipped with suction slots for boundary-layer control showed higher lifts attainable than by other means. Rolling distance and take-off speed were greatly reduced by application of suction at take-off. During flight, application of suction doubled the maximum wing-lift otherwise obtainable, without influencing unfavorably other flight characteristics. Nor did a sudden cessation of suction result in dangerous flight behavior. L. Landweber, USA

2745. C. B. Smith, A solution for the lift and drag of airfoils with air inlets and suction slots, J. aero. Sci. 16, 581–589 (Oct. 1949).

Representing suction or pressure slots by sinks or sources (point singularities) located on contour of airfoil, lift, and drag of airfoil in two-dimensional incompressible potential flow are calculated by means of complex contour integration. It appears that the drag is equal to $\rho U_0 \Sigma m_n$ (ρ is density; U_0 free-stream velocity; m_n volume flow from source) and the lift to $\rho U_0 \Gamma$ (Γ circulation), with $\Gamma = -\Sigma m_n \tan \frac{\epsilon - \beta_n}{2} + 4\pi R U_0 \sin (\alpha + \epsilon)$ (ϵ is polar angle of trailing edge point on circular image with radius R of airfoil contour, after conformal transformation; β_n is polar angle of nth source on this image; α is angle of attack). Tests in the UAC laminar-flow wind tunnel confirm these results as soon as quantity of sucked air surpasses amount necessary for boundary-layer removal. J. H. Greidanus, Holland

2746. J. Calvin Lovell and Stanley Lipson, An analysis of the effect of lift-drag ratio and stalling speed on landing-flare characteristics, Nat. adv. Comm. Aero. tech. Note 1930, 34 pp. (Sept. 1949).

The increased application of airfoils and wing plan forms designed specifically from high-speed considerations has resulted in airplanes having relatively low maximum lift coefficients, low lift-drag ratios, and high wing loadings. These design trends increase stalling speed and gliding angle of airplanes; hence, an adverse effect on landing maneuver is indicated. Calculations of landing-flare paths of a series of hypothetical airplanes having systematically varying characteristics have been made to provide information concerning effects of above parameters on landing-flare characteristics. Charts are presented which indicate effects of current design trends on landing-flare velocity and distance requirements and which permit rapid estimation of flare characteristics of any airplane having an approximately constant lift-drag ratio during landing flare. A method for predicting landing-flare characteristics of an airplane having a variable lift-drag ratio during flare is given in an appendix. Application of analysis is demonstrated by determination of effects of high-lift devices on landing characteristics of three transonic wing configurations.

Airplanes landing at low lift-drag ratios will have relatively high sinking speeds at start of flare and at an altitude of 50 feet, even if stalling speeds are relatively low. Airplanes which have relatively low lift-drag ratios and high stalling speeds will require power-off landing flares to start at relatively high altitudes. These results indicate that power-off landings of some airplanes designed specifically from high-speed considerations may not be

feasible at conventional airports. Airplanes having high stalling speeds will require relatively large horizontal distances for either power-on or power-off landing flares.

Analysis includes detailed calculations resulting from a study of landing-flare characteristics of wings having 37- and 48-deg sweepback, as well as those of triangular wings. Sinking speed and other general characteristics during flare are listed.

E. Arthur Bonney, USA

2747. **G. A. Mokrzycki, Calculation of the disturbed motion of an aircraft,** Proc. seventh int. Congr. appl. Mech. 2, part II, 548–553 (1948).

2748. **Harvard Lomax and Max A. Heaslet, Damping-in-roll calculations for slender swept-back wings and slender wing-body combinations,** Nat. adv. Comm. Aero. tech. Note 1950, 32 pp. (Sept. 1949).

The damping-in-roll parameter C_{l_p} is calculated theoretically for case of triangular wings on cylindrical bodies in order to extend the knowledge of wing-body interference into field of lateral stability. Parameter C_{l_p} is also found for a class of wings with sweptback plan forms, and it is shown how C_{l_p} can be calculated for swept wings with arbitrary trailing edges, or how trailing edges can be calculated from prescribed span load distributions. Two examples are given which indicate how other cases can be calculated numerically.

Analysis is based on usual assumptions of linearized compressible-flow theory together with added assumption that $(1 - M_0^2)\phi_{xx}$ is small compared with ϕ_{1yy} and ϕ_{1zz}, where ϕ is the perturbation velocity potential, M_0 is free-stream Mach number, and x is free-stream flow direction. Under these assumptions the equation for compressible flow, either subsonic or supersonic, becomes $\phi_{yy} + \phi_{zz} = 0$. Accuracy of results is tested for case of a triangular wing by a comparison with exact solution of linearized equation.

J. S. Isenberg, USA

2749. **Carlo Ferrari, Interference between wing and body at supersonic speeds—note on wind-tunnel results and addendum to calculations,** J. aero. Sci. 16, no. 9, 542–546 (Sept. 1949).

Note is supplementary to an earlier paper by same author (Rev 1, 1268). It is stated that original calculations contain an error concerning the boundary condition assumed in region where body covers the wing. A method of successive approximation is described; it is a modification of original procedure. Corrected boundary condition involves a discontinuous upwash distribution along span of wing; resulting analytical difficulties are discussed in some detail. Numerical example given in earlier paper is amended and result compared with experiment. The calculation of the total pressure increment on wing involves a distinct "contour" effect which is due to wing being situated within radial pressure field of body.

A. Robinson, England

2750. **Jack D. Stephenson, The effects of aerodynamic brakes upon the speed characteristics of airplanes,** Nat. adv. Comm. Aero. tech. Note 1939, 34 pp. (Sept. 1949).

Report is concerned with problem of relating drag increases due to aerodynamic brakes to control of forward speed. Requirements which must be met in order for brakes to provide necessary control over forward speed are discussed for various flight conditions under which they may be used. Equations relating speed and altitude are presented for several cases in which certain simplifying assumptions are made. For these cases, formulas and graphs in report furnish a means of quickly computing longitudinal speed variations, dive angles, and rates of descent for airplanes having known drag characteristics. For cases in which the simplifying assumptions do not apply, it is indicated that a satisfactory solution, which takes into account all possible variables (such as atmospheric density, drag coefficient, and flight-path angle), can be obtained by a step-by-step method of calculation. Graphs are presented to reduce time required for step-by-step calculations. Example calculations, which show each step in detail, illustrate use of graphs and formulas.

Increases in drag coefficient that are characteristic of several types of wing and fuselage aerodynamic brakes, which have been tested in wind tunnels or in flight, are summarized in report. Effect of Mach number on drag coefficient and effect of partial brake deflection are included where such data are available. Problems of buffeting, of changes in stability, of aerodynamic loads, and of changes in trim, which may arise when a particular brake is used on an airplane, are not considered. Discussion of practical use of brakes and various designs are included.

E. Arthur Bonney, USA

2751. **B. Filzek, Investigations on the stability, oscillation, and stress conditions of airplanes with tab control; first partial report—derivation of the equations of motion and their general solutions,** Nat. adv. Comm. Aero. tech. Memo. no. 1197, 49 pp. (Sept. 1949).

Longitudinal equations of motion are written for an airplane controlled longitudinally by means of a servotab on the elevator. The equations are set up by LaGrange's energy method. Described servotab linkage is connected to a lever which is mounted on center line of elevator hinge, but not mechanically connected to elevator except by a spring. A serious error is made by including both tab push rod forces and spring forces in summation of elevator hinge moments. The equations are solved, and it is shown how one may compute transient responses, including various initial conditions, and also frequency responses, including conditions necessary for a permanent steady-state.

Graham Campbell, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 2586)

2752. **J. Dorr, Span effect in flutter-calculations. Comparative calculations on a circular tail unit** (in French), Rech. aéro. Paris no. 11, 57–63 (Sept.-Oct. 1949).

Results of binary flutter analyses of vertical empennage of Junkers 288 airplane are presented to show great influence of three-dimensional aerodynamic effects in flutter of surfaces having very low geometric aspect ratio. Investigation is based upon an actual occurrence of flutter, in which predominant motions of empennage were lateral translation and rotation about a vertical axis in the plane of symmetry. Calculations employing three-dimensional aerodynamic theory yield critical speeds which are 50 to 80 per cent greater than those determined by strip theory. Actual flutter speed is not mentioned. Effects of three-dimensional flow are estimated by means of theory of oscillating airfoil of circular plan form due to Th. Schade and K. Krienes.

M. J. Turner, USA

2753. **H. M. Lyon, Aerodynamical derivatives of flexural-torsional flutter of a wing of finite span,** Rep. Memo. aero. Res. Counc. Lond. no. 1900, 26 pp. (1949).

This and the following review relate to a report completed in 1939 but only recently released. They represent an attack on problem of finite oscillating airfoil in a steady incompressible stream. Normal displacement of wing at any point is found in terms of a distribution of vorticity involving mn undetermined

constants. By assigning the displacements in a vibration mode at *mn* points, it is possible to solve for vorticity distribution and aerodynamic derivatives. Stephen H. Crandall, USA

2754. W. P. Jones and Sylvia W. Skan, Calculations of derivatives for a wing of finite span, Rep. Memo. aero. Res. Counc. Lond. no. 1900, 34 pp. (1949).

Report is concerned principally with evaluating the integrals involved in Lyon's report (see preceding review). For one frequency and one aspect ratio, calculations are carried through to obtain aerodynamic derivatives for a rigid airfoil pitching about midchord. Result for this case is not significantly different from that predicted by strip theory. Stephen H. Crandall, USA

2755. C. Scruton, W. G. Raymer, and D. V. Dunsdon, Experimental determination of the aerodynamic derivatives for flexural-aileron flutter of B. A. C. wing type 167, Aero. Res. Counc. Rep. & Mem. R. & M. no. 2373, 15 pp. (1950).

In order to carry out measurements indicated by title, a rigid $^1/_{20}$ scale model of title wing was hinged to wind-tunnel wall along a chordwise line at its root section. A rigid aileron was hinged to wing. Angular deflections of aileron about its hinge line and of wing about its root constituted two degrees of freedom. Air speed of tunnel was varied from 20 to 60 fps and the derivatives determined for a range of reduced frequencies. For direct derivatives, unconcerned degree of freedom was restrained while response of other degree of freedom to a sinusoidal exciting force of varying frequency was measured with and without aerodynamic effects, which were thereby isolated. For compound or coupled derivatives, the element associated with one degree of freedom was oscillated by direct displacement, while moments arising therefrom in the other degree of freedom were determined either by direct measurement or by calculation from a measured response. Apparent mass or aerodynamic inertias were not measured.

Results are compared with those of an analysis based on two-dimensional oscillating wing theory. Comparison indicates that experimental values involving aileron were 60 to 70 per cent of theoretical, while those involving bending alone were somewhat better, being 80 per cent of theoretical. Paul A. Libby, USA

2756. A. E. van de Vooren and D. J. Hofsommer, Binary aileron-tab flutter, Nat. Luchtlab. Amsterdam Rep. F.52, 44 pp. + 3 tab. + 10 fig. (Sept. 1949).

Results are reported of an investigation of effects of important parameters on stability of aileron-tab systems. Tab may be a spring tab, balance tab, or trim tab. Investigated system consists of a fixed rectangular wing with an aileron and a tab. Combinations of aileron moment of inertia, tab moment of inertia, and tab static moment have been calculated which lead to stability at all speeds. Results are presented in diagram form.

R. L. Bisplinghoff, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 2687, 2698)

2757. J. Zborzek, Ground effect on the lifting rotor, Aero. Res. Counc. Rep. & Mem. R. & M. no. 2347, 8 pp. (1950).

An analysis is made of results of model tests in hovering conducted in Göttingen (1944) and by the NACA (1941). Rotors with different numbers and shapes of blades, pitch settings, and rotor speeds are used. Conclusions are presented by curves showing η_H/η_∞ versus $C_{T\infty}/\sigma_{0.7}$ for various values of a/D, where η_H is efficiency proportional to $C_T^{3/2}/C_Q$, C_T and C_Q thrust and torque coefficients, $\sigma_{0.7}$ solidity at 0.7 radius, a/D ratio of rotor height above ground to rotor diameter, and subscript ∞ denotes values for infinite a/D. The curves are also given in form of T/T_∞ vs. a/D for various values of $C_{T\infty}/\sigma_{0.7}$, and T/T_∞ vs. $C_{T\infty}/\sigma_{0.7}$ for various a/D. Curves clearly show relative increase of thrust (at constant power) due to proximity of rotor to ground. Results of a flight test are shown to agree with presented curves. Morris Morduchow, USA

2758. V. M. Astafier, Differential equations of gas turbines with an infinite number of blades and their integrals (in Russian), Doklady Akad. Nauk SSSR 68, 449–452 (Sept. 1949).

This short note deals with representation of flow of fluid in rotating apparatus with an infinite number of blades. Note is based on work of Stodola in 1907. L. M. Tichvinsky, USA

2759. G. C. I. Gardiner and J. Mullin, The design of propellers, J. roy. aero. Soc. 53, 745–756 (Aug. 1949).

Paper reviews British propeller-design practice as currently applied to transport aircraft. The following general topics of propeller design are discussed: Propeller efficiency and thrust; factors governing choice of propeller diameter; noise and rotational tip speed; blade area; number of blades; blade-airfoil sections; propeller weight; restrictions imposed by necessity for some degree of standardization; steady stresses; vibration stresses; materials; speed control; feathering; reverse-pitch control; and future trends. General discussion in paper avoids consideration of any one particular type of propeller, although the few detailed comments are based on hydraulically operated propellers. Frank L. Wattendorf, USA

2760. W. Richter, On a formula for axial flow compressors with uniform pressure (in German), Ingen.-Arch. 17, no. 1/2, 88–93 (1949).

Paper presents a theoretical analysis of flow through a Schicht-type blower or compressor stage which is characterized by maintaining a constant static pressure through rotor. Constant pressure is achieved by suitable decrease of free cross-sectional area through rotor. A relationship is derived between the blade tangent and cross-sectional area which interrelates blade curvature with curvature of housing or hub.

This investigation is based on one-dimensional flow theory. Accordingly, influence on flow of centrifugal forces caused by curved contour of housing or hub is neglected. In addition, effects of tangential velocity components on radial distribution of velocity and static pressure are not taken into consideration.

Frank L. Wattendorf, USA

Flow and Flight Test Techniques

(See also Revs. 2713, 2730, 2787, 2788, 2829)

2761. J. H. Preston, The effect of a wind tunnel fan on irregularities in the velocity distribution, Aero. Res. Coun. Rep. & Mem. R. & M. no. 2307, 4 pp. (1950).

Use of a gauze or a windmill is a known method of improving velocity distribution in a wind tunnel. In paper, the action of a fan, generating pressure rise across its disk, is investigated theoretically. It is shown that a fan also reduces flow deviations in a similar manner. G. V. R. Rao, USA

2762. Garrett Birkhoff and Thomas E. Caywood, Fluid flow patterns, J. appl. Phys. 20, 646–659 (July 1949).

One of the most difficult tasks in fluid mechanics is to measure

magnitude and direction of velocity in a field of flow. Paper describes an important technique, originally due to L. B. Slichter, which is particularly well adapted to study of phenomena accompanying entry of solid missiles into liquids. In one form, experiment consists in taking, on a single photographic plate, two flash pictures of a field containing missile and a system of small air bubbles rising under gravity with a velocity of order 3 to 8 cm/sec; diameter of bubbles is of order of 0.5 mm and time interval Δt between flashes is a few milliseconds. Clearly, if vector displacements, Δ_x, of various bubbles in known times, Δt, are measured, velocity at various points in field can be determined, both in magnitude and in direction. In an alternative method, air bubbles are replaced by droplets of hexane (density 0.69 gm/cc).

Several interesting photographs are included; they relate to vertical entry of spheres, spheroids and wedges, oblique entry of spheres, and one is an excellent example of ricochet of an aluminum sphere at an impact angle of 7 deg. Observed velocity fields are compared with those predicted by theory, and agreement is satisfactory.

In spite of its simplicity, method is subject to three corrections: (1) "Wall" effect due to finite size of experimental tank, (2) rise of bubbles in time interval Δt due to buoyancy, and (3) differences in displacements of bubbles and surrounding water under sudden accelerations, due to density differences ("virtual mass" effect). Magnitudes of these corrections are investigated theoretically and experimentally, and authors conclude that method is well adapted for velocities lying between 5 and 100 fps, created by objects several inches or more in diameter; at other speeds or with smaller objects, results become difficult to interpret. An appendix gives a brief account of some measurements made in course of the work on drag coefficients of bubbles and distortion of bubbles in pressure fields.

Some photographs lack detail because of bad reproduction and fig. 19 has been printed upside down. R. M. Davies, Wales

2763. **Johs Krutzsch, A new apparatus for measuring densities of liquids** (in German), Chem.-Ing.-Tech. 21, 467–468 (Dec. 1949).

A "thread" of liquid to be investigated is sucked into a vertical capillary tube, open at lower end. Upper end is connected to a U-tube containing a liquid of known density. Required density is then found by measuring heights of liquid columns. Method has advantage that only 1 or 2 cc's of liquid are required. Maximum possible error is stated to be about 2 per cent.

W. A. Mair, England

2764. **P. Mandl and J. R. Pounder, Wind tunnel interference on rolling moment of a rotating wing,** Nat. Res. Counc. Canada Aero. Rep. MA-216, 36 pp. (June 1949).

Authors determine variation of rolling-moment coefficient of a rotating wing in uniform flow due to interference in a wind tunnel of circular cross section. Method is based on hypothesis that for calculation of downwash angle one can assume same spanwise circulation in tunnel as in free air. Determination of such downwash for a given circulation is done by a method similar to one followed by H. Reissner in his study of flow past a multibladed propeller in free air. When compared with the analysis of Evans who, when calculating downwash, supposes the trailing vortex rectilinear, present report has the merit of taking into account helical nature of the free vortex. Compared with method suggested by Schmieden, it has the advantage that results are valid whatever the wing span to tunnel diameter ratio. Reviewer notes that this advantage is lessened by above-mentioned hypothesis about law of variation of circulation along the wing span.

The correction factor for rolling coefficient is derived for a closed and an open tunnel, for several wing span to tunnel diameter ratios over a range of values of $\omega s/V$ between 0 and 0.4 (ω is angular velocity of wing; s wing semispan; V free-stream velocity), and for wings of several taper ratios, both in case of rectangular and of elliptic antisymmetric loading. Numerous appendixes follow, in one of which (by theory of images) correction factor is obtained for a rectangular tunnel under Evans' hypothesis (rectilinear free vortex). Carlo Ferrari, Italy

2765. **D. L. Ellis, Industrial and wind tunnels,** J. roy. aero. Soc. 53, 797–818 (Aug. 1949).

Paper discusses design and application of wind tunnels for industrial use. A conventional 9 × 7-ft low-subsonic tunnel is described in detail. A shorter description of a smaller transonic tunnel, powered with a turbojet unit acting as an ejector pump, is also presented. J. M. Wild, USA

2766. **M. S. Macovsky, W. L. Stracke, and J. V. Wehausen, Measurement of intensity of turbulence in water by dye diffusion,** David Taylor Model Basin Rep. 700, 23 pp. (July 1949).

Exploratory experiments were made to develop a method for measuring transverse turbulence intensity in a water flow. Experiments were made in a water channel downstream of a grid. A dye solution was discharged through an injector, and samples of water were extracted at various distances from injection point. Average dye concentration of samples was measured with an electrophotometer. Standard deviations of cross-stream distribution of the concentrations were determined, and transverse turbulence intensity was computed. A comparison was made with hot-wire measurements. It is shown that longitudinal turbulence intensity measured with the latter is sensibly equal to the transverse turbulence intensity.

In an appendix the influence of finite size of sampling tube upon measured dye concentrations is computed. Cases of round and square samplers are investigated, assuming that the average dye concentration in a cross-stream section can be represented by a Gaussian error surface. F. N. Frenkiel, USA

2767. **Edward Wenk, Jr., A diaphragm-type gage for measuring low pressures in fluids,** Nav. Dept. David Taylor Mod. Basin, Rep. 665, 15 pp. (Jan. 1950).

Gage employs a circular diaphragm with fixed edges with two SR-4 strain gages mounted to measure radial strain. Characteristics of gage: Natural frequency, 7500 cps; sensitivity, 40 microinches per in. per lb per sq in.; linearity at 70 F, 0.5 per cent full scale; variation in sensitivity from 20 to 120 F, 10 per cent; temperature sensitivity, 0.5 microinches per in. per deg F. Excellent discussion of factors affecting sensitivity, temperature effects, etc. More recent developments (at M.I.T. as yet unreported) employ a membrane-type diaphragm permitting an increase in natural frequency without loss of sensitivity.

Robert J. Hansen, USA

2768. **Eli Ossofsky, Constant temperature operation of the hot-wire anemometer at high frequency,** Rev. sci. Instrum. 19, 881–889 (Dec. 1948).

An extensive analysis for design of a constant temperature hot-wire anemometer developed at the NACA Lewis Flight Propulsion Laboratory has produced a method for obtaining stable operation and high frequency response. The circuit, consisting of a Wheatstone bridge and a d-c amplifier, is analyzed by feedback amplifier theory. Transimpedance of bridge and time lag of resistance-capacitance coupled multistage amplifier are related to dynamic feedback factor. Stability limits must be

established such that stable frequency range of amplifier is several times that of instrument as a whole. Range and attenuation are interrelated for the simple cutoff and Bode cutoff. Circuit diagram for an instrument covering frequencies from zero to 30,000 cps is shown. Anemometer characteristics are computed for other frequency ranges. Because of noise limitations, constant temperature instruments are found five times less sensitive to small-scale turbulence than constant current hot-wire anemometers. J. R. Weske, USA

2769. Gene J. Delio, Glennon V. Schwent, and Richard S. Cesaro, Transient behavior of lumped-constant systems for sensing gas pressures, Nat. adv. Comm. Aero. tech. Note 1988, 35 pp. (Dec. 1949).

Paper analyzes a pressure-sensing system which may be part of a pneumatic measuring device. The system, consisting of a tube and a reservoir, is identified, under specified approximations, with a system of lumped-constant mass, resistance, and stiffness. The indicial response of an actual system is compared with transients of electrical theory, and approximation appears to be satisfactory. Interest of paper lies mainly in expressions for dependence of (a) undamped natural frequency upon temperature, and (b) damping ratio upon viscosity, temperature, and pressure. An example is given in which increased resistance to flow leads to a desired increase in damping ratio of system without affecting undamped natural frequency.

P. Le Corbeiller, USA

2770. F. D. Werner, An investigation of the possible use of the glow discharge as a means for measuring air flow characteristics, Rev. sci. Instrum. 21, 61–68 (Jan 1950).

Author shows characteristics of glow discharge (current vs. voltage) for steady-state conditions of pressure, temperature, humidity, and velocity. He concludes that phenomenon may be of use for measurement of transient pressure waves, sound waves, and turbulence. No conclusive evidence is presented for transient temperature effects. Reviewer believes that method might give difficulty for precision transient pressure measurements. Robert J. Hansen, USA

Thermodynamics

(See also Revs. 2782, 2784, 2786)

2771. R. O. King, W. A. Wallace, and E. J. Durand, The oxidation, ignition, and detonation of fuel vapors and gases. IX. The cause of the reversal of the antiknock property of rich hydrocarbon-air mixtures, Canad. J. Res. Sec. F 27, 307–310 (Aug. 1949).

Experiments described show that conditions can be set in which a reversal of antiknock effect of enriching a hydrocarbon-air mixture can be obtained in an unsupercharged engine. Reversal is of importance in respect to supercharged aero engines, in which it may occur before mixture strength is increased to value required for development of maximum power. Experimental results, considered in the light of nuclear theory of detonation, indicate that "reversal" occurs when rate of formation of finely divided carbon by pyrolysis of fuel provides a proknock effect greater than can be offset by antiknock effect of products of high-temperature heterogeneous oxidation reaction.

Test data on unsupercharged CFR engine with high octane commercial fuel, leaded and unleaded, indicate "reversal" to occur in range of mixtures 80% to 100% rich. It is expected that reversal would occur at mixtures leaner than this in supercharged engines of heavy duty and aero types.

W. M. Rohsenow, USA

2772. E. Bartholomé, The flame velocity in stationary flames (in German), Naturwissenschaften 36: no. 6, 171–175; no. 7, 206–213 (Aug. 1949).

Flames can be used directly in such technical processes as production of acetylene if flame velocity, W_f, can be accurately controlled. Furthermore, flames provide one of the few means of studying homogeneous, high-temperature chemical reactions. Author is, therefore, interested in studying factors that control W_f. Experimental data for a considerable number of fuel-oxidant combinations is analyzed with following conclusions: (a) W_f is determined by flame temperature only, for a given class of fuels, if this temperature is not too high; (b) for high-temperature flames, hydrogen atom concentration H in flame fixes W_f. (b) is in disagreement with the view of Hoare and Linnett (Rev 2, 1168) although latter agrees that W_f correlates well with H.

To support analysis of experimental data, thermal reaction theory of flames is developed and applied particularly to CH_4–O_2 flames. Although author's theory is less complete than recent analyses of Corner and of Hirschfelder [J. Chem. Phys. 17, p. 1076 (1949)], it is an improvement over older thermal theories and leads easily to predictions that can be tested against the experimental data. Bruce L. Hicks, USA

2773. K. Steinbuch, Computation of piston temperatures (in German), Ingen.-Arch. 17, 353–362 (1949).

The difficult problem of temperature distribution in piston of a reciprocating engine is discussed. Transient effects are eliminated by limiting problem to time averages. Frictional effects are ignored. On basis of several simplifying assumptions a scheme of calculations, involving an iteration process, is given for temperature distribution in head of piston, in neighborhood of piston rings, and in cylindrical walls. It is assumed that heat transferred to head of piston from hot combustion gases is removed in following three ways: (1) Heat flows from ring zone to cylinder; (2) heat flows from shaft zone to cylinder; and (3) heat flows from head of piston to exhaust gases.

A mathematical treatment of heat transfers is first presented. Then the calculated temperature distribution is compared with some measured temperatures in an actual engine. Agreement between the two distributions is good, considering assumptions made to get an engineering answer. Joseph Kaye, USA

2774. Harold Grad, On the kinetic theory of rarefied gases, Commun. pure appl. Math. II, 331–407 (1949).

Paper presents an essentially new method for determining continuum flow equations from fundamental Maxwell-Boltzmann equation of kinetic theory. Instead of classical Hilbert-Enskog perturbation solution which leads to successively more complex equations involving just five dependent variables (density, temperature, and three velocity components) for increasingly rarified gases, author proposes an approximation based on successively higher moments of velocity distribution function which is expanded in terms of Hermite polynomials. Number of dependent variables and number as well as complexity of equations for successive approximations increases from one approximation to the next. First significant approximation, which involves both viscous stresses and heat transfer, is the third approximation which leads to 20 equations in 20 independent variables. Stresses, for example, are entirely independent variables unrelated, except implicitly in the differential equations, to gradients of even arbitrarily high order of velocity, density, and temperature. Relaxation times appear which are characteristic of adjustment of stresses and heat flow to quasisteady states if given initial disturbances. For such quasisteady states and for small gradients in velocities and temperature, the additional differential equations

may be identified with classical Navier-Stokes and Newton expressions for stresses and heat flux to a first approximation, and in second approximation to terms similar to, but not identical with, Burnett terms. For a rarefied gas, stresses, for example, depart from Navier-Stokes expressions only for a nonuniform temperature distribution. Characteristics of the differential equations are used to determine number of boundary conditions, and Hermite polynomial expansion is used to determine form of boundary conditions. Terms analogous to velocity slip and temperature jump are obtained. S. A. Schaaf, USA

2775. **G. Leibfried and F. Kaempffer, A simple method for calculating sums of energy-states** (in German), Z. Physik. 124, 444–449 (1948).

Calculation of distribution function of an ideal Bose-Einstein gas by means of grand partition sum involves two summations: (1) Over-all occupation numbers of a single energy state, and (2) over-all energy states. Inverting the usual order, the authors carry out the second summation first. Results admit a formal interpretation as representing a mixture of Boltzmann particles. Calculation is carried out explicitly for a system of linear oscillators obeying Bose-Einstein statistics.

Courtesy of Mathematical Reviews L. Tisza, USA

2776. **Max Kohler, Treatment of non-equilibrium phenomena by means of an extremal principle** (in German), Z. Physik 124, 772–789 (1948).

D. Enskog [Thesis, Uppsala (1917)] has shown that the fundamental kinetic equation of kinetic theory of gases can be written in form of a variational principle. Correct distribution function in stationary state is characterized by maximum value of rate of increase of entropy per unit volume and unit time. Here, distribution function has to satisfy certain auxiliary conditions. Author shows that same quantity is a minimum if constancy of heat current and viscous stress are chosen as auxiliary conditions. The two extremum principles are equivalent. They can be used as a basis for a Ritz approximation procedure. According to chosen function sets, one obtains Enskog or Burnett [Proc. Lond. math. Soc. (2) 39, 385–430 (1935)] equations. Method is applied to quantum statistical problem of metal electrons.

Courtesy of Mathematical Reviews L. Tisza, USA

2777. **R. Vichnievsky and N. Manson, Application of the modern thermodynamic methods for the study of the work cycles in the heat engines**, Proc. seventh int. Congr. appl. Mech. 3, 211–225 (1948).

Authors investigate thermodynamic analysis of such heat engines as gas turbine, turbojet, and reciprocating engines. They point out the unavoidable errors obtained when using standard entropy, enthalpy, internal energy, and temperature charts which are constructed for reference fuels only.

L. M. Tichvinsky, USA

2778. **Fritz Sauter, On the equation of state of rarefied real gases** (in German), Annal. Phys. Leipzig 6, 59–66 (Sept. 1949).

The van der Waal equation of state is known to be more accurate than the simple ideal gas law, primarily because it corrects for space occupied by the gas molecules and for forces exerted between molecules. Various methods, usually complex and difficult to comprehend physically, have been proposed for calculating magnitudes of these corrections. After reviewing these, author concludes that the only logical method is one based on the known Boltzmann's law, from which it follows that average kinetic energy in each degree of freedom can be expressed as a function of temperature. With only this law and two simple laws of statistical mechanics, magnitudes of the corrections are determined by an elementary analysis, much simpler than an analysis by the common virial method. It is a consequence of Boltzmann's law that, because of finite size of gas molecules, density and pressure are increased in immediate vicinity of a bounding wall. When attractive forces exist between gas molecules and wall (a case which cannot be treated directly by the virial method), this method shows that gas pressure is independent of magnitude of attractive forces, a result known from experience to be true.

C. W. Smith, USA

2779. **J. E. Verschaffelt, On the thermodynamics of irreversible processes** (in French), Physica Hague 15, 489–502 (July 1949).

The thermomechanical problem of diffusion has been analyzed by means of first and second law of thermodynamics, making use of a principle of superposition first introduced by Rayleigh and later reformulated by Onsager. Results are in agreement with Onsager's principle of reciprocity. Serge Gratch, USA

2780. **William Band and Lothar Meyer, Non-equilibrium states in helium II**, Phys. Rev. 76, 417–423 (Aug. 1, 1949).

Propagation of second sound in He II is studied in terms of two-fluid model developed by Tisza. It is shown that an apparent violation of the second law of thermodynamics can be resolved by assuming that propagation of second sound occurs in absence of thermal equilibrium between superfluid and normal component of He II. Tendency of assembly to return irreversibly to state of thermal equilibrium within a certain relaxation time provides a consistent account of observed limitations of superfluidity at critical velocity, and of heat conductivity for wide capillaries. Relaxation times are calculated according to this picture from heat-conductivity measurements and from Pellam's direct attenuation measurements in second sound. The times from these two distinct experimental sources are shown to be identical within accuracy of available data.

Serge Gratch, USA

2781. **J. De Boer, Development of probability densities in power series of the density**, Physica Hague 15, 680–688 (Sept. 1949).

Paper is an extension of author's previous one [same source, 15, p. 680 (1949)], and develops the probability density for an arbitrary number of molecules in power series of the density. The power series are substituted to give general expressions for equation of state. Terms proportional to density are evaluated and shown to be the same as those obtained by method of statistical thermodynamics. Keith J. Laidler, USA

Heat and Mass Transfer

(See also Revs. 2571, 2693, 2730, 2773, 2779)

2782. **D. Geist and U. Dehlinger, Heat conduction during change of phase** (in German), Z. Naturforsch. A 4a, 415–423 (Sept. 1949).

Authors investigate problem of heat conduction due to a moving plane heat source. Analysis is developed from known solution of temperature distribution due to a stationary plane source, and is represented finally by an integral equation in $v(t)$, the source velocity. Two cases are considered in detail: (a) When $v(t)$ is constant, and (b) when $v(t)$ is a linear function of temperature at the spatial position of source.

The calculations are applied to solidification of molten substances. It is shown that under conditions of subcooling, con-

densation nuclei form along a boundary which moves through molten substance at a finite velocity. Liberated heat of fusion raises the temperature of subcooled substance to equilibrium melting temperature. It is finally shown that the laminated structure of metal crystals can have no direct effect on heat-conduction process.

G. M. Lilley, England

2783. R. D. Hoyle, Unsteady heat flow in a large irregular solid, Proc. seventh int. Congr. appl. Mech. 3, 64–78 (1948).

In a nonrevolving steam turbine rotor of 25-in. diam, heated by steam (400 psi, 700 F), the distribution of temperature in a cross section through impulse wheel is measured. Most severe gradients—over 100 F per in.—are found about two min after turning on steam. In 90 min the steady state has not been nearly approached. Calculated temperatures, by use of Maclaurin's theory (for same boundary conditions), checked very well. Calculation of temperature stresses is planned.

Otakar Maštovský, Czechoslovakia

2784. L. Miller, Temperature effects in the diffusion of real gases (in German), Z. Naturforsch. vol. A 4a, 262–265 (July 1949).

Experimental measurements of the transient temperature field occurring with diffusion of two gases into each other is reported. In diffusion of hydrogen and nitrogen, after 0.5 sec a maximum temperature rise of 0.93 C occurs in hydrogen about 4 mm from original interface, and after 1 sec a maximum temperature drop of 1.95 C occurs in nitrogen about 8.5 mm from interface. The full temperature field is given in this case, while maximum temperature increases and decreases are listed for a number of other pairs of gases. A brief discussion is given of possibilities of correlation with thermodynamic heat of mixing. Test results are also mentioned for diffusion into free air.

Newman A. Hall, USA

2785. E. F. M. van der Held and F. G. van Drunen, A method of measuring the thermal conductivity of liquids, Physica Hague 15, 865–881 (Oct. 1949); abridged: Proc. seventh int. Congr. appl. Mech. 3, 79–90 (1948).

Method often employed for measuring thermal conductivity λ of liquids and gases consists of heating a straight platinum wire stretched in center of a cylindrical tube filled with the test material. Steady-state temperature of the wire must then be measured. Main difficulty in this method is to avoid convection currents within liquid. Authors hope to avoid this difficulty by using the analogous unsteady method with a very short time of measurement. The constant heat production q in wire causes an unsteady cylindrical temperature field in liquid. They observe the rise of temperature ϑ at surface of wire during first 0 to 50 sec. Taking difference in temperature $\vartheta_2 - \vartheta_1$ at two instants t_2 and t_1, they obtain a formula $\vartheta_2 - \vartheta_1 = (q/4\pi\lambda)ln(t_2/t_1)$. A practical apparatus for carrying out the tests in this manner is described.

Some corrections must be considered with this method because of the influence of thickness of the heater wire and of diameter of tube, and because of the uncertainty in fixing the time of beginning of experiment. As a test of the method, authors have undertaken to measure thermal conductivity of water. The result agrees well with the best modern measurements of this quantity. The conductivity of 24 liquids near 20 C is dealt with (butanol, methanol, aniline, benzene, acetone, pentanol, nitric acid, acetic acid, hydrogen chloride, etc.). The accuracy of those values is estimated at 2 per cent.

Walter Fritz, Germany

2786. W. H. McAdams, W. E. Kennel, J. N. Addoms, C. S. Minden, and C. M. Gamel, High densities of heat flux from metal to water, NEPA Heat Transf. Lect., NEPA Div., Oak Ridge, Tenn., 1, 17–41 (1948).

Paper reviews work on high densities of heat flux from metal to water, including recent work done at M.I.T. Paper calls attention to work of Moscicki and Broder (1926) which has been overlooked. They found that heat flux can be increased about seven times by using subcooled water rather than water at boiling point.

Recently some very interesting work has been done at M.I.T. Special apparatus has been built so as to vary water velocity as well as temperature. These experiments give information on the mechanics of boiling. Values ten times that obtained with water at boiling point and flux densities of 4,700,000 are reported.

R. M. Wingren, USA

2787. L. M. K. Boelter and W. H. Sharp, An investigation of aircraft heaters XXXII—Measurement of thermal conductivity of air and of exhaust gases between 50 and 900 F, Nat. adv. Comm. Aero. tech. Note 1912, 39 pp. (July 1949).

Thermal conductivity of air at temperatures ranging from 50 to 900 F and thermal conductivity of gasoline-engine exhaust gases at temperatures ranging from 250 to 900 F have been measured and are presented in graphical form. Details of apparatus and method of calibration are described. It is shown that thermal conductivity of exhaust gases is approximately same as that of air when operating fuel-air ratio of engine is low, but tends to increase as compared to thermal conductivity of air when fuel-air ratio is increased.

John E. Goldberg, USA

2788. G. A. Hawkins, The measurement of rapidly changing temperatures, Nucl. Energy Power Aircr. Heat Transf. Lect. I, 47–58 (Dec. 1948).

Author provides an informative introduction to problems of fabricating and applying thermocouples suitable for measuring rapidly fluctuating temperatures in gases and solids. Specific designs of thermocouples for measuring temperatures at bore surfaces of guns are given with recommended assembly techniques. A means for determining magnitude of error existing in thermocouple measurements of high-velocity gas stream temperatures is presented in terms of gas velocity and thermocouple recovery factor (beta) which is determined experimentally. Two methods for fabricating small wire (0.001 to 0.0001-in. diam) thermocouples are described. One employs an oil-covered mercury bath and battery for arc welding the junction. A more successful method for small wires employs a carbon electrode welder. High-speed recording and indicating apparatus using amplifier, oscilloscope, and rotating drum camera are described. Difficulties encountered when using d-c amplifiers are enumerated and a satisfactory a-c chopper amplifier is described. Schematic diagram of amplifier is presented together with circuit utilized for time-position identification on photographic paper. Bibliography of worth-while references is appended.

Edward Cartotto, USA

Acoustics

(See also Revs. 2581, 2759, 2780)

2789. A. de Rosen, The silencing of test houses for turbojet engines, Aircr. Engng. 21, 236–241 (Aug. 1949).

Author considers following problem: How shall a structure for housing a turbojet testing facility be designed in order to assure that residual noise 50 yd from test house shall not seriously reduce effectiveness of a loudspeaker system operating in vicinity of listening area? Given answer is qualitative in nature.

An interesting review is given of information that must be available to the design engineer in order to arrive at needed attenuation. Important factors are: Main noise sources (air intake and exhaust); distribution of acoustic energy at source as a function of frequency; and distribution at listening point which will assure an articulation index of better than 40 per cent.

The greater portion of required attenuation must be associated with exhaust silencer. Author advocates, for this purpose, a splitter-type exhaust having a distribution of bends and expansion chambers along its length. An equal volume of fresh air is entrained in exhaust in order to effect a cooling of about 200 C. A similar treatment is recommended for intake. Procedures for applying these ideas are not given, but a recent installation at Le Bourget incorporating them has proven very successful. Curves and results obtained are presented. Horace M. Trent, USA

2790. **J. Maulard, On the speed of sound in the air at low pressures** (in French), C. R. Acad. Sci. Paris, 229, 25–26 (July 1949).

By measuring velocity of sound in air in a tube 27.43 m long, having an inside diam of 8 cm, effect of pressure on velocity is determined. At one end a metallic plate generates sound which is picked up at other end by a microphone, and output is put on vertical plate of a cathode ray oscillograph. Timing is accomplished by a relaxation oscillator controlled by a standard frequency. Warren P. Mason, USA

2791. **Leo L. Beranek and George A. Work, Sound transmission through multiple structures containing flexible blankets,** J. acoust. Soc. Amer. 21, 419–428 (July 1949).

Authors treat general case of transmission of normally incident sound through a structure which consists, in general, of an impervious septum, air space, flexible blanket, impervious septum, and a flexible blanket. The impervious septa are assumed to be pure mass reactances, while blankets and air space have distributed parameters so that they are characterized by a complex propagation constant and a characteristic impedance. At interfaces the solutions are linked by appropriate impedance requirements. Finally, the ratio of pressure transmitted into a ρc termination to input pressure is computed. By setting different parameters equal to zero, a large number of panels are considered. The propagation constant and characteristic impedance of blankets may be obtained from specific acoustic resistivity, density, structure factor, volume coefficient of elasticity, and porosity of the blanket.

Experimentally, measurements were made on 18 × 18-in. samples. Good agreement occurred with theory. Classical-weight-law attenuations for impervious septa were not obtained unless they were damped by thin mica sheets. Composite sandwiches made of mica-damped aluminum sheets gave results in accordance with the weight law, presumably due to damping of flexural waves. According to reviewer's work [J. Res. nat. Bur. Stands. 42, Rep. 1998, p. 605 (1949)] this would lead one to suspect that other than normally incident sound waves were present, since no flexural effects should occur for normally incident in phase waves. A comparison is made between reverberant measurements and normal incidence calculations on an office partition, consisting of two sheets of metal with an absorbing blanket. Reverberant measurements give attenuations which are too low, compared to calculations, by as much as 70 db at 10,000 cps. Reviewer feels these differences arise both from random-incidence sound field, and from fact that metal sheets are not pure mass reactances but also have stiffness due to flexural effects, and, in addition, are dissipative. Albert London, USA

2792. **Alfred Leitner, Diffraction of sound by a circular disk,** J. acoust. Soc. Amer. 21, 331–334 (July 1949).

Diffraction fields of a thin circular disk are plotted according to equations developed by Bouwkamp and others. These equations give solutions to wave function in oblate spheroidal coordinates, and are, therefore, of special application to bodies of oblate spheroidal form. With one exception, theoretical curves are based on assumption of an infinitely thin rigid disk which is the limiting oblate spheroid of zero "radial" coordinate. In the one exception this coordinate is given a small value so as to approximate a disk of small but finite thickness. Theoretical values are compared with experimental values for a number of ratios of diameter to wave length, and found to be in good agreement. Gerald Pickett, USA

2793. **Italo Barducci, Response of some telephonic receivers measured with an artificial ear** (in Italian), Ric. Sci. 19, 689–695 (July 1949).

Frequency response of six commercial telephone receivers is presented and discussed. Their response has been measured by using an "artificial ear" consisting of a cavity, a condenser microphone membrane, and three holes to simulate the leaks. Results, given in figures and tables, are discussed by author in practical terms. Leslie S. G. Kovasznay, USA

2794. **A. A. Kharkevich, Power horns and Stokes polynomials** (in Russian), Doklady Akad. Nauk SSSR 68, 685–688 (1949).

An acoustic horn is discussed which has cross-sectional area varying as x^{2n} where x is distance along the axis of the horn. The pressure p in the horn is given by an expression involving Bessel functions of $\omega x/c$ which reduces, for integer n, to the form $p = F(kx) \exp(-ikx)$. Here $k = \omega/c$ and F is a polynomial in $1/kx$. The characteristic impedance of the horn is then a rational function of ω and has an equivalent circuit of ladder type.
Courtesy of Mathematical Reviews E. N. Gilbert, USA

2795. **Winston E. Kock and F. K. Harvey, Refracting sound waves,** J. acoust. Soc. Amer. 21, 471–487 (Sept. 1949).

Several mechanical structures for refracting sound waves are described and pictured. These act either by increasing the effective density of medium traversed by sound, or by providing path delays. Assemblies of disks, spheres, and strips were used for the first kind and parallel slanted plates for second. Equations and design procedures are outlined. Lenses and prisms were constructed with diameters from 6 to 30 in.; they were tested successfully in neighborhood of 10 kc. A divergent lens to reduce excessive high-frequency directivity of a conventional loud speaker is also described. A. O. Williams, Jr., USA

2796. **Virginia Griffing and Francis E. Fox, Theory of ultrasonic intensity gain due to concave reflectors,** J. acoust. Soc. Amer. 21, 348–351 (July 1949).

An ultrasonic beam can be focused by a lens or concave mirror, with consequent large gains in intensity in focal plane. Gain is limited by diffraction, as in optics. Assuming plane waves, in a beam of either circular or rectangular cross section, under conditions of Fraunhofer diffraction, flux through any given area of focal plane can be computed. Accordingly, intensity gain averaged over such an area can be computed from known optical solutions. For beams of circular section (whether limited by mirror or by some other agency), result is expressible in simple Bessel functions. Results for a rectangular beam are in terms of trigonometric functions and the sine integral. A numerical table is given to facilitate such calculations. Method can be extended to beams screened by a strip or a straight edge, and to a circular

receiving area in a rectangular beam. [There is a minor typographical slip in Eq. (10): $J_1(z)$ should be squared. Reviewer wishes to emphasize that basic assumption of plane waves suffering pure Fraunhofer diffraction may sometimes be oversimplified.]
A. O. Williams, Jr., USA

2797. Louis Fein, Ultrasonic radiation from curved quartz crystals, J. acoust. Soc. Amer. 21, 511–516 (Sept. 1949).

In order to obtain many physical, chemical, and biological effects, high sound intensities, often accompanied by cavitation, are required. Simplest method for attaining high sound intensities is the focusing radiator made from crystalline quartz or ceramic barium titanate. Efficiencies of such radiators are investigated in present paper by electrical impedance methods and by acoustic radiation-pressure methods.

By measuring electrical resistance of the crystal at resonance in air and in liquid, the various sources of loss can be obtained and efficiency determined for converting from electrical into acoustic radiation power. Four radiators are investigated which have radii of curvature from 4 cm to ∞ (flat plate). Efficiencies calculated from electrical measurements run from 80 to 95 per cent.

Acoustic pressure measurements were made by using a crystal probe, and these patterns as a function of position check previous calculations by A. L. Williams. By integrating these measurements over space surrounding radiator, efficiency of conversion can be calculated and these efficiencies run from 20 per cent for most highly curved crystal to 74 per cent. In all cases they are lower than efficiencies calculated from electrical measurements. Low efficiency of highly curved radiator is attributed to fact that crystal does not vibrate as a piston.
W. P. Mason, USA

2798. H. Billing, Sound sources in rectilinear motion (in German), Z. angew. Math. Mech. 29, 267–274 (Sept. 1949).

Velocity potential of a source moving with subsonic velocity and having a strength which exhibits a harmonic time dependence is calculated from linearized equations using an attack due essentially to H. G. Küssner [Luftfahrtforschung 17, 370–378 (1940)]. Results are applied to a study of the pressure field of a circular cylinder due to harmonically varying circulation associated with formation of the von Kármán vortex street. Disagreement between the author's results and previously known experimental measurements is attributed to presence of strong turbulence in incident flow in latter case.
J. W. Miles, USA

2799. H. T. O'Neil, Theory of focusing radiators, J. acoust. Soc. Amer. 21, 516–526 (Sept. 1949).

Author considers problem of sound field produced by a concave spherical radiator in which all elements of surface execute radial simple harmonic motions having equal amplitudes and phases. A number of approximations are made which are clearly delineated. These are: (1) That radius, a, of boundary circle of radiator is large compared to both depth of concave surface, h, and to wave length of emitted sound; (2) that medium is devoid of absorption and acts in a strictly linear fashion; and (3) that velocity potential at any point is given by the integral

$$\psi = \int\int \frac{u}{2\pi s} \exp(-iks)dS$$

where $k = 2\pi$ divided by wave length, dS an element of surface of radiator, u normal velocity at surface, and s distance from dS to point of observation. Assumption (3) is rigorously correct only if (2) holds and if each elementary surface element emits strictly hemispherical waves. Keeping the approximations in mind, author works out expressions for sound field near focal point and in focal plane. It is shown that the ratio of intensity at center of curvature to average intensity at radiating surface is nearly equal to $(kh)^2$, and that intensity in central part of focal plane, at an angle θ from the axis, is approximately proportional to $(2/ka \sin\theta)[J_1(ka \sin\theta)]$.
Horace M. Trent, USA

2800. O. O. Gruenz, Jr., and L. O. Schott, Extraction and portrayal of pitch of speech sounds, J. acoust. Soc. Amer. 21, 487–495 (Sept. 1949).

A method for showing how pitch of the voice varies as one speaks is described. It employs a combination of gain control, double detection, voiced sound selection, unvoiced sound exclusion, and a means for counting the fundamental vibrations in the voiced sound intervals. Electric signals from a microphone that picks up sound waves are passed into the automatic gain control, which brings levels of practically all sounds to same value. Signals are modified through several stages so as to increase the basic low frequency components which determine pitch and diminish higher frequency components. This results in a spike wave, frequency of waves giving the pitch.

Reliable indications of pitch were obtained over a range of 100 to 600 cycles for a wide variety of voices. Pitch-color and pitch-time systems for visual portrayal of speech patterns are described. Circuit diagrams are given, and operation of each unit is described.
Gerald Pickett, USA

2801. P. Liénard, Sound field produced by a point source moving uniformly and rectilinearly in a perfect fluid, especially in the case of supersonic velocity, Rech. aéro. Paris 10, 43–51 (July-Aug. 1949).

If a sound source moves at supersonic speed, the pressure at each point inside the cone of sound is shown to be equal to the sum of two pressures, differing as to amplitude, phase, and direction of propagation. To an observer moving with source, these two pressures oscillate at same frequency. Author analyzes loci of the maxima and minima of pressure, of equal mean-square pressure, and of equal phase. He shows that an observer, moving in relation to source of sound, for every emitted frequency generally receives two different variable frequencies of sound. In certain cases, latter are reduced to one frequency, variable or constant.

It is shown that meridians of the cone are asymptotes of hyperbolic meridians of the surfaces of alternating maxima and minima of pressure inside cone. It is also shown that pressures in these hyperbolic surfaces decrease as the distance from their apexes increases. Derivation of equations presumes familiarity with mathematics of theory of sound. A number of diagrams illustrate theoretical findings.

Author points out that, for practical application of his method, a source of sound of very low frequency should be used, but he does not go into any details concerning use of his analysis for measurement of supersonic speeds of airplanes.
O. R. Wikander, USA

2802. Osman K. Mawardi, On the propagation of sound waves in narrow conduits, J. acoust. Soc. Amer. 21, 482–486 (Sept. 1949).

Propagation of sound in narrow conduits of arbitrary shape is considered. A wave equation similar only in form to one first given by Helmholtz is written down which includes inertial and dissipation terms resulting from viscous effects. Velocity of sound in conduit is shown to depend primarily on a parameter $f^{1/2}S/P$, where f is frequency, S cross-sectional area, and P perimeter, and only secondarily on shape of conduit. At low frequencies ($f^{1/2}S/P < 0.10$) velocity corresponds to Newton's isothermal value, while at high frequencies ($f^{1/2}S/P > 10$) it assumes

Laplace's adiabatic value. Narrow conduit is then replaced by a leaky electrical transmission-line analog, which has the characteristic impedance and propagation constant obtained from wave equation. Input acoustical impedance of a large tube into which straight round wires have been forced is then computed. The interstitial channel comprising the conduit in this case is that formed by three cylinders, two of which are tangent to each other at a time, thus forming a curved triangular shape for conduit. Measurements of acoustic impedance agree with theory although considerable scatter between the two was experienced. A calculation of flow resistance was also made and it was found necessary to assume that interstitial channel was that formed by four cylinders, tangent two at a time, to get agreement with theory. Necessity for this assumption is attributed to fact that because of twist in wires only 1860 out of theoretical 2300 could be packed in tube.

Albert London, USA

Ballistics, Detonics (Explosions)

(See also Rev. 2820)

2803. **E. Jones, The ignition of solid explosive media by hot wires,** Proc. roy. Soc. Lond. Ser. A 198, no. 1055, 523–539 (Sept. 1949).

Paper presents an investigation of critical thermal energy required to cause ignition of solid explosives by electrically heated fine resistance wires embedded in the medium. The energy equation, formulated on basis of the investigation, indicates that ignition is a purely thermal process and that the critical factor is temperature.

A. Petroff, USA

2804. **F. P. Bowden and A. Yoffe, High temperature sources and ignition of explosives** (in Swedish), Tekn. Tidskr. 79, 791–796 (Oct. 29, 1949).

For a long time it has been a known fact that many explosives, liquid or solid, may be detonated by impact and by friction, but mechanism and conditions which determine whether detonation will occur or not have as yet not been clearly understood. It has now become evident that detonation is started from small local sources of high temperature. There are three ways of obtaining these sources: (1) Through friction between the explosive and grits or other solid objects, (2) through adiabatic compression of small gas bubbles entrapped in explosive, and (3) through viscous flow of liquid explosive. In first case, it is shown that melting point of grit must not be less than 480 C in order to have necessary local temperature rise. In second case, influences of entrapped gas, initial pressure, as well as energy of impact have been examined.

Chemical analysis of gaseous products resulting from the explosion indicates a fast thermal decomposition of explosive rather than a real detonation, which may develop at a later stage.

W. Weibull, Sweden

Soil Mechanics, Seepage

(See also Rev. 2671)

2805. **M. E. de Beer, Some examples of application of research methods in Belgium for the solution of foundation problems** (in French), Ann. Inst. Tech. Bat. Trav. Publ. (A) 105, Sols et Fond., 24 pp. (Dec. 1949).

Methods described are cone penetration tests for field investigations, limit equilibrium method for design of shallow footings, and triaxial and consolidation laboratory tests. Examples of use of these standard methods are cited in design of particular foundations.

Eben Vey, USA

2806. **John McNamee, Seepage into a sheeted excavation,** Géotechnique Lond. 1, 229–241 (Dec. 1949).

Review is given of elements of theory of seepage and limitations on applicability to flow in soils as actually encountered. A number of examples of seepage into excavations protected by sheet piles are treated. Two-dimensional flow is assumed and flow patterns are obtained by relaxation for various combinations of depth of sheet piling, depth of excavation, and thickness of pervious layer. A series of curves is presented for limiting case of incipient piping at surface of excavation.

It is noteworthy that an analytical solution to this problem has already been obtained by methods of conformal transformation, although no general curves were prepared [V. C. Koslov (in Russian), Izvestia Acad. Sci., Tech. Sci. Sect. no. 6 (1939)].

John S. McNown, USA

2807. **R. R. Proctor, Laboratory soil compaction methods, penetration resistance measurements, and the indicated saturated penetration resistance,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 242–247 (June 1948).

Author describes various "standard" soil-compaction procedures that others have adopted since his original method was presented in 1933. He points out that his original compactive effort consisted of "firm blows" of tamper, rather than free fall commonly used. Although author's concept of laboratory compaction might be influenced by personal element, he states that no difficulties have been encountered in 17 years of its use. In paper the various laboratory compactive efforts are compared on an energy basis of ft-lb/cu ft of compacted soil, and the values are further correlated with energy exerted in field compaction, as measured by draw-bar pull of tractor pulling the rollers.

Author compares strengths and densities attained with so-called values of 90 and 95% of compaction, showing that these percentages are not based on corresponding reductions in strength of soil from "optimum" condition, but may actually reflect much greater losses in strength. He advocates use of indicated saturated penetration resistance, called ISPR, which is determined by measuring penetration resistance of soil as molded, and then extrapolating to penetration resistance that soil would have if it were saturated. Data presented show a definite relationship between compaction energy, density, shear strength, and ISPR. Method of determining ISPR may be considered empirical, which is not objectionable in itself, but it may not give an accurate picture of strength of soil when actually saturated. However, author has used method successfully for a number of years in control of fill construction.

Woodland G. Shockley, USA

2808. **A. Casagrande and W. L. Shannon, Stress-deformation and strength characteristics of soils under dynamic loads,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 29–34 (1948).

Authors describe apparatus which they developed and results which they obtained in investigating dynamic strength and deformation characteristic of soils.

Dynamic tests are defined as those in which test specimen is caused to fail within a time-loading interval of the order of a fraction of a second. Conventional tests, in which time-loading interval is of the order of minutes or longer, are called static tests. Purpose of investigations is to determine effect of time of loading in static and dynamic time-loading ranges on unconfined and triaxial tests performed on cohesive and cohesionless soils.

Tests were performed on one type of clean medium sand and on three types of inorganic clay having liquid limits ranging from 37 to 64 and plastic limits ranging from 20 to 27. Tentative conclusions drawn from these tests are: (1) *Strength of clay* increases with decreasing time of loading, transient strength for fastest

tests in this investigation being from 1.5 to 2 times static strength. Percentage increase in strength is dependent on static strength. Samples with low static strength had a greater percentage increase than those with high static strength. Increase in strength due to time of loading is independent of method of testing. (2) *Strength of sand* increases only slightly with decreasing time of loading. Maximum increase for fastest tests was about 10%. (3) *Modulus of deformation of clay* for tested transient tests was about twice that for static tests. R. E. Fadum, USA

2809. R. Grammel, Deceleration of rotationally symmetric bodies dropped upon a sand bed (in German), Ingen.-Arch. 17, 219–222 (1949).

Author postulates following phenomenological law: For deceleration of a body falling upon a bed of sand, decrease in momentum in an element of time dt is proportional to mass of sand displaced in that same element of time. Based on this assumed law, expressions in nondimensional form are derived for acceleration, velocity, and displacement as functions of time for several different types of bodies of revolution. In case of a conical body, it is found that these nondimensional expressions are independent of cone angle, mass of falling body, striking velocity, and constitution of sand. Remarkably close agreement is found between results calculated by this simple theory and some experimental results obtained by R. Feldtkeller. Dana Young, USA

2810. W. Bernatzik, Contribution to the problem of the seepage pressure in electro-osmosis, Proc. sec. int. Conf. Soil Mech. Found. Engng. 7, 65–66 (June 1948).

Two identical samples of quartz powder were placed in a vertical glass tube 2.7 cm in diameter. Samples were saturated and separated by a space in the tube containing only water. An electric potential was applied across lower sample only and, at the same time, a hydraulic pressure was applied to bottom of tube. Latter was increased until top sample failed at critical hydraulic gradient. Bottom sample showed no sign of such failure even under increased potential. Test was repeated with top sample switched out. A plot of velocity of flow vs. hydraulic gradient in latter case gives a straight line parallel to line obtained when only hydraulic pressure is applied. Author claims these results show that electroosmosis produces no pore water stresses, intergranular stresses, nor forces on electrodes. E. Vey, USA

2811. Edgar Schultze, Soil pressures and ground failure (in German), Abh. Bodenmech. Grundbau, pp. 51–58; Berlin, Erich Schmidt Verlag (1948).

Paper begins with a brief survey of current theories concerning load capacity of a semi-infinite cohesive soil mass under uniformly distributed normal surface forces applied to an infinitely long strip (two-dimensional problem). Prandtl's slip-line method and its modifications by Buisman and Raes are reviewed. Paper then develops a modified version of Pihera's method based on principal stress trajectories and offers several applications of it. Graphical procedures are outlined for nonuniform and inclined surface forces. Oscar Hoffman, USA

2812. Karl Terzaghi, Soil moisture, in *Hydrology*, ed. by O. E. Meinzer, New York, Dover Publications, 331–363 (1949).

Author presents equations derived by a theoretical approach to determination of moisture and capillary action in soils. He makes the following points: (1) Actual height of capillary rise in soils cannot be computed by ordinary theoretical equations based on grain size. This is chiefly due to effect of nature of impurities surrounding soil particles. (2) No satisfactory theoretical equations are available for computing height of capillary rise in relation to time. This is due to an increase in rate at which coefficient of permeability decreases as width of voids approaches thickness of adsorbed water layer surrounding soil particles. (3) Laboratory drainage tests on water-holding power of soils cannot yield results comparable to field results, because the field degree of saturation is the result of intermittent recharge by rain combined with gravity drainage. However, pending more field data, author recommends use of centrifuge method for laboratory-drainage tests. Then, knowing depth of the full-size soil layer, he suggests that degree of saturation for field conditions is the same order of magnitude as degree of saturation at which laboratory discharge, computed for full-size layer in field, is equal to average recharge. Eben Vey, USA

2813. Alfred Basch, Geometric rules governing subsoil water flow, Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 280–285 (1948).

2814. A. W. Skempton, The $\phi = 0$ analysis of stability and its theoretical basis, Proc. sec. int. Conf. Soil Mech. Found. Engng. 1, 72–78 (June 1948).

Classical experimental results showing existence of an angle of shear-resistance ϕ equal to zero for clays and silts are reviewed, as are the stability problems whose resolutions are based on assumption of $\phi = 0$. Author then presents limitations of the $\phi = 0$ analysis, insisting that its application can only be made when there is no consolidation under action of the solicitations and when the materials are effectively saturated, noting that for some particular saturated silts ϕ is not zero. He insists, especially, on fact that $\phi = 0$ does not lead to a zero value of the angle of internal friction; thus, the classical $\phi = 0$ analysis does not give, in general, the actual sliding surface. Nor, theoretically, does the application of that analysis furnish a correct value of the factor of safety. He concludes, nevertheless, that, in spite of these difficulties in the interpretation of the phenomena, experience shows that the $\phi = 0$ analysis is reliable, having great practical value in stability computations. Manuel Rocha, Portugal

2815. H. Lorenz, Prevention and stabilization of slope slides (in German), Bautechnik. 25, no. 11, 243–246 (Nov. 1948).

2816. Nabor Carrillo, Influence of the rigidity of a rectangular slab on the settlements and on the distribution of elastic soil reactions, Proc. sec. int. Conf. Soil. Mech. Found. Engng. 7, 28–30 (June 1948).

First part of paper deals with problem of an infinitely long rectangular slab subjected to uniform load, q, and supported by a semi-infinite elastic solid. After a brief discussion of the two limiting cases, (a) infinitely rigid slab and (b) infinitely flexible slab, paper proposes following semi-empirical formula for maximum bending moment in slab for intermediate cases of finite flexural rigidity: $M = ql^2/(29.278 + 3.399E_s/E)(l/h)^3$ where l is slab width, h its thickness, E Young's modulus for slab, E_s Young's modulus for supporting solid. In second part of paper an approximate method is proposed for determining the maximum moment in rectangular slabs of finite length.

Oscar Hoffman, USA

2817. Henri Marcus, Soil reaction to slow deformations caused by consolidation and to strains caused by pile loads, Proc. sec. int. Conf. Soil Mech. Found. Engng. 7, 1–11 (June 1948).

Assuming elastic conditions, author sets up equations of equilibrium and stress-strain relationship for condition of axial symmetrical stress in a body of soil (case I). He assumes a type of deformation for the radial and vertical displacements. Expres-

sions for the state of stress produced by slow consolidation of a compressible soil underlain by stiff substrata and subjected to a load at the surface are next derived (case II). Deformations are given in terms of the deflection increase of top surface and functions obtained from case. I. Finally, the case of pile loads is considered (case III). By combining results of cases I and II, it is shown that load which pile can carry by skin friction is proportional to shear modulus of surrounding soil and inversely proportional to compressibility of supporting soil. For soft substrata, paths of one set of principal stresses transmit the skin load by arch action into surrounding soil. Circular stress at top of substrata increases from pile outward to outermost principal stress trajectory. Point of minimum vertical pressure is also point of maximum circular pressure, and the soil acts like a ring absorbing the thrust of spherical shells. Radial pressure created by initial stresses due to pouring or driving the pile tends to expand the soil rings and reduce earth pressure in horizontal direction. By writing initial stresses in terms of coefficients which depend on natural horizontal pressure, pile section, nature of soil, and compaction produced by pouring or driving, author arrives at allowable pile loads to produce tensile stresses in soil equal to soil cohesion. He further assumes soil strength recovery to the extent that the active earth pressure in circular direction exists at time of loading. Preload stresses then are superimposed on load stresses to give complete state of stress. Condition for stability requires that the term expressing shear strength in active pressure formula not be exceeded; thus it is shown how much soil readjustment increases its resistance. Experimental results are cited to demonstrate that load for failure according to approximate theory occurs long before actual critical pile-bearing capacity is reached.

Eben Vey, USA

2818. **R. J. Hank and F. H. Scrivner, Some numerical solutions of stresses in two or three layered systems,** Highway Res. Bd., Proc. 28th ann. Meeting, 28, 457–468 (1948).

2819. **Reginald A. Barron, Transformations for flow net construction,** Proc. sec. int. Conf. Soil Mech. Found Engng. 7, 166–168 (June 1948).

Paper presents extensions to method of transforming soil sections that are nonhomogeneous and anisotropic to sections that are nonhomogeneous and isotropic so that flow nets may be developed or electrical model tests be performed on transformed sections.

From author's summary

2820. **Benjamin F. Howell, Ground vibrations near explosions,** Bull. seism. Soc. Amer. 39, 285–310 (Oct. 1949).

In seismic prospecting, use is normally made only of compressional waves reflected or refracted from subsurface elastic discontinuities. Other types of waves, often observed on records, cause interference and complicate interpretation. Author has made an experimental study of all types of ground motion observable within 3500 meters of an explosion, using three-component detectors and a recording system giving flat response from $3\frac{1}{2}$ to 80 cps. Experiments were conducted in an area where formations from surface to 100-ft depth appear unconsolidated.

All of the observed wave types have been previously described in the literature. These include Rayleigh waves, coupled and hydrodynamic waves of the kind found by Leet, and possibly Love waves. Time-distance plots and particle-motion diagrams are presented for various waves. Body waves are discussed briefly. An effort is made to relate coupled waves and hydrodynamic waves to waves predicted by classical theory. It is suggested that coupled waves are longitudinally polarized refracted shear waves, although no mechanism is proposed by which their properties can be predicted from geometry and elastic constants of near-surface formations. Author was unable to identify observed hydrodynamic waves with any established wave type. An attempt to explain the Rayleigh wave characteristics by Stoneley's theory for a liquid layer (the air and low-speed surface zone) overlying a semi-infinite elastic medium yields inconclusive results. The difficulties are attributed to poorly defined elastic characteristics of unconsolidated sediments.

Reviewer believes that the experimental data should be reexamined in light of recent theoretical work by Cagniard, Pekeris, and Press and Ewing. Dispersion of Rayleigh waves, apparent on the records but not discussed in paper, might provide a valuable aid to interpretation based on normal mode theory.

Milton B. Dobrin, USA

Micromeritics

(See Rev. 2809)

Geophysics, Meteorology, Oceanography

(See also Rev. 2594)

2821. **F. J. Sines, Production and study of ocean waves in the laboratory,** Dock & Harbour Authority XXX, no. 351, 280–281 (Jan. 1950).

At the University of Washington in Seattle, a new type of wave machine has been installed in the 170-ft wave channel. Machine duplicates theoretical trochoidal wave properties with a flexible stainless-steel plate perpendicular to direction of wave travel and actuated by synchronized adjustable rotating arms. Variable wave heights are obtained by adjusting the plate stroke at surface. Wave reflection in channel is reduced by concrete-cylinder wave absorbers mounted below water surface. Present and future studies include: Comparing the generated deep-water wave to its theoretical properties; shallow-water waves using a sloping bottom; stability of beaches subjected to wave action; and practicability of building deep-water breakwaters.

W. Wuest, Germany

2822. **E. Palmen, Origin and structure of high-level cyclones south of the maximum westerlies,** Tellus 1, no. 1, 22–31 (1949).

The very important study of southward transport of cold air, and related northward transport of warm air, across the belt of middle-latitude westerlies is undertaken from point of view of its actual mechanism. An element of cold air is traced as it pushes southward and becomes cut off from remaining cold air to north. The life history of this cold cyclonic vortex, as it maintains itself south of the belt of maximum westerlies, is then studied by means of contour and temperature charts at several pressures, cross sections showing the tropopause and frontal deformations. Some elementary theoretical considerations are given. Author finds a well-marked, definite transition between polar and tropical air in the form of a sharp isotherm concentration at levels as high as 500 and 400 mb. It is the collapsing (by spreading out below of the central regions of cold air dome more rapidly than southern portion) which isolates the cold cyclonic vortex on upper level charts. Although extremely pronounced just below the tropopause, this vortex commonly weakens both in the stratosphere and in the very low levels, being commonly entirely lacking on the surface maps. Weakening of circulation near ground is stated to be at least partially responsible for long lives of these vortexes since they are only slowly weakened by friction. It would also have been interesting to see a study of dying phases of these vortexes, and how they must eventually impart many of their properties to their new surroundings.

Careful and complete case studies such as the present can be extremely valuable in studying mechanism of heat transfer in general circulation of atmosphere, and it is hoped that they may be continued in conjunction with, and as a stimulus to theoretical studies on these problems. Joanne Starr Malkus, USA

2823. J. G. Charney and A. Eliassen, A numerical method for predicting the perturbations of the middle latitude westerlies, Tellus 1, 38–54 (May 1949).

As a first step in a program for numerical weather prediction, planetary waves in a zonal current are studied for the case of a barotropic atmosphere. In contradistinction to theory, these long waves are often observed to be quasistationary in westerlies of middle latitudes. It is shown that this discrepancy can be removed by taking shape of ground (profile of continents) and surface friction into account. From this, numerical method is developed for predicting change of height of the 500-mb pressure surface at a fixed latitude. In deriving the simplified forecast equation from vorticity equation (averaged with respect to pressure in vertical direction), all solutions giving fast-moving disturbances are suppressed by means of quasigeostrophic approximation [see J. G. Charney, On the scale of atmospheric motions, Geof. Publ. XVII, no. 2 (1948)]. Introducing approximate expressions for vertical wind component near ground and for surface friction, a forecast equation is derived which can be solved numerically. Fron this, the quasistationary long wave disturbances in atmosphere appear to be forced perturbations due to geographically fixed centers. Results of six successive forecasts, each for a period of one day, are presented and compared with observations. Agreement is satisfactory.

Horst Merbt, Germany

2824. Masao Siotani and Giiti Yamamoto, Atmospheric turbulence over a large city, I, II (in Japanese), J. met. Soc. Japan 27: 67–77; 111–115 (Mar. and Apr. 1949).

Wind speed was recorded for 10 min by five hot-wire anemometers mounted on a wireless tower at Central Meteorological Observatory located in center of Tokyo. Heights of anemometers were 60, 55, 45, 35, and 25 m above ground. From oscillographic records authors calculate mean wind speed, root-mean-square of turbulent fluctuations, coefficient of eddy viscosity, time and space correlations, spectral distribution of turbulent energy, etc.

Vertical distribution of mean speed seems to follow logarithmic law, roughness height Z_0 amounting to 4 to 7 m, while heights of neighboring buildings were 10 to 20 m. Integral scale of turbulence deduced from time correlation curve ranges from 3 m for weak winds to 30 m for strong winds at a height of 60 m. Vertical distributions of scale and other statistical characteristics of turbulence, however, fail to show any systematic regularity.

Itiro Tani, Japan

2825. Kyoichi Takeda, The vertical distribution of wind velocity near the earth's surface (in Japanese), J. met. Soc. Japan 27: 333–341; 363–370 (Nov. and Dec. 1949).

Temperature and wind velocity were observed at four heights (0.5, 1, 2, and 5 m above ground) on shrubby plain at foot of Mt. Fuji. Distribution of wind velocity u is found to be expressed by a simple formula $u = a \log (z/z_0)$, where z is height above ground, and a and z_0 are constants. It appears that a is independent of vertical distribution of temperature but z_0 increases as stability increases. The simple formula gives a good explanation for observations of Best and Heywood, but fails to describe Hellmann's data. Formula also leads to relation

$$K_a = K_s(1 + \beta R_i),$$

where K_a and K_s are eddy viscosity in adiabatic and stratified atmospheres respectively, R_i is Richardson number, and β is Rossby constant. Itiro Tani, Japan

2826. Masao Tiotani and Giiti Yamamoto, Atmospheric turbulence near the ground, I, II (in Japanese), J. met. Soc. Japan 27: 73–77; 219–224 (Mar. and July 1949).

Turbulence of wind near ground was investigated by same method as used for previous observation in free atmosphere (see REV 2, 951). In first report, hot-wire anemometers were placed at several heights above ground on calm summer days at cultivated plain of Ibaraki. Intensity of turbulence is less than 20% at 5 m high and increases downwards. Scale of turbulence and eddy viscosity increase upwards, the magnitudes at 10 m height being about 4 m and 2×10^4 cm²/s, respectively. Second report refers to observation made in a narrow basin surrounded by hills. Pattern of wind structure appears to differ considerably from that on a flat plain. Itiro Tani, Japan

2827. G. R. Goldsbrough, The tides in oceans on a rotating globe, Proc. roy. Soc. Lond. Ser. A 200, 191–200 (Jan. 1950).

Previous solutions of problem of tides in oceans bounded by meridians have involved much arithmetical work. Paper gives an analytical form of solution for luni-solar tide-producing potential (K_2), under assumption of a special law of depth. Equations are integrated by the Galerkin method. Henry Stommel, USA

2828. E. T. Eady, Long waves and cyclone waves, Tellus 1, 33–52 (Aug. 1949).

Present paper aims at developing from first principles a simplified quantitative theory of initial stages of development of wave cyclones and long waves. Complexity of problem requires significant approximations. Thus, internal friction is neglected and Coriolis parameter taken constant. Other simplifications by omitting terms in dynamic equations are justified by a detailed analysis of result of approximative theory.

Disturbances of steady baroclinic large-scale atmospheric motion are studied first, and complete solutions satisfying all the relevant simultaneous differential equations and boundary conditions show that these simple states of motion are almost invariably unstable.

An arbitrary disturbance (corresponding to some inhomogeneity of an actual system) may be regarded as analyzed into components of a certain simple type, some of which grow exponentially with time. In all examined cases there exists one particular component which grows faster than any other and becomes dominant. Characteristic disturbances (forms of breakdown) of certain types of initial system (approximating those observed in practice) have for maximum growth-rate approximately same wave length as observed long waves. For smaller values of static stability (as in large cloud masses) and smaller vertical extents, we obtain "dominant" wave lengths of order of magnitude of observed extratropical wave-cyclones. Possibility of instability is also shown by an energy analysis. Finally, implications regarding ultimate limitations of weather forecasting are discussed. Walter Wuest, Germany

2829. Georg Weinblum and Walter Block, Stereophotogrammetric wave photographs, David Taylor Model Basin Transl. no 204, 76 pp. (1949).

Report consists essentially of 10 stereophotogrammetric wave photographs taken from the *MS San Francisco*, a German freighter, during a research voyage in the North Atlantic in December

1934. Among these is the highest wave ever measured quantitatively, over 60 ft high. Photogrammetric methods of plotting contours is described in detail. A list of additional instruments for measuring movements of ship is included. In analysis of records, authors appear to be unaware of considerable progress made in this subject during the last ten years in England and the USA.
Walter H. Munk, USA

2830. **Walter Wuest, Contribution to the generation of water waves by wind** (in German), Z. angew. Math. Mech. 29, 239–252 (July and Aug. 1949).

Author reviews briefly the observational data on effect of natural and artificial winds on a horizontal body of water. These data show a slow movement of water near surface produced by surface friction. Above a certain critical speed of wind, surface waves appear. Their wave length depends on wind velocity and on length of path of wind over water surface. Critical wind velocity observed in controlled experiments with artificial winds is about 2.5 m/sec, and critical wave length about 1.7 cm.

Most of paper is devoted to author's theory that waves are produced by instability of water surface and, in particular, to computation of stability when flow of air and water near boundary is laminar and confined to thin boundary layers.

The familiar fourth-order linearized differential equation for disturbance stream function is used with boundary conditions appropriate to the problem. A first computation is made with wind velocity constant with height (infinitely thin boundary layer in air) and with water at rest (surface friction neglected). A critical wind velocity of 1.6 m/sec is obtained with a wave length of 3 cm. A second computation is made with linear velocity distributions in the boundary layers. In this computation some observational data on mean velocities and boundary-layer thicknesses are used. It is found that water boundary layer has no appreciable influence on results. A third computation is made to consider an air boundary layer with Blasius-type velocity distribution.

Results of second and third theoretical computations are: (1) Third computation (Blasius distribution) gives no fundamental changes in results as compared with second computation (linear distribution). (2) Wave length of waves first appearing is markedly dependent on boundary-layer thickness and increases with increasing thickness. Most easily excited waves have wave lengths of 2 to 4 cm, and they are amplified at wind velocities as low as 0.7 m/sec. Discrepancy with experimentally observed value of critical velocity is attributed to turbulent boundary layers present in experiments.
Hugh L. Dryden, USA

2831. **G. E. R. Deacon, Waves and swell,** Quart. J. roy. met. Soc. 75, 227–238 (July 1949).

A comprehensive survey is presented of recent British and U.S. experimental and theoretical investigations on dependence of waves and swell on atmospheric conditions. Any storm will create a large number of different wave trains, their velocity being proportional to period. Thus at greater distances, long waves will arrive first. Highest amplitudes are observed at periods of about $^3/_4$ that of longest swell. From simple cases, where only one wave-creating storm was present, it was possible to determine reliable data on frequency and energy distribution in these wave trains. Observed group velocities are in good agreement with theory. Velocity of longest swell was found to be nearly the same as the gradient wind velocity. All waves are subject to various changes in shape, direction, and velocity near shore, due to coastal conditions and tides, but methods now available (theoretical as well as empirical) should allow prediction of swell and surface phenomena in most cases.
Horst Merbt, Germany

2832. **H. R. Seiwell, The principles of time series analyses applied to ocean wave data,** Proc. nat. Acad. Sci. Wash. 35, 518–528 (Sept. 1949).

British scientists have developed methods of analysis of ocean waves based on Fourier series, and find that the sea-surface roughness pattern is composed of component wave trains which travel from their area of generation with group velocities appropriate to their frequency. Author's autocorrelation analysis of wave records from Bermuda and Cuttyhunk islands leads him to conclusion that sea-surface pattern is not such an interference pattern composed of many frequencies (which he ascribes to application of Fourier methods to a finite amount of data), but represents the superposition of random frequencies (local sea) on a *single* sinusoidal component (sea swell). He suggests that ocean itself acts as a filter which passes only certain fundamental frequencies (sea swell, with periods of the order of 10 sec), those frequencies changing from day to day; furthermore he finds that these frequencies of the sea swell equal mean frequency of random local sea. Whereas he has demonstrated usefulness of autocorrelation as a tool in analysis of ocean waves, physical interpretation of his results should be further elaborated.
Walter H. Nunk, USA

2833. **Ralph Stair, Seasonal variation of ozone at Washington, D. C.,** J. Res. nat. Bur. Stands. 43, 209–220 (1949).

Any observations of the ozonosphere are of extreme importance to the meteorologist, because of the hypothesis that this ozone layer may indeed form the link between solar variations and alterations in the weather patterns of the lower atmosphere.

Present paper on ozone measurements, however, suffers the same severe limitation as most of its predecessors: namely, observations were made from the ground, and, therefore, to compute the thickness of ozone present, it was necessary to assume a spectral energy curve for the sun in the region 3000 to 3400 Å. Although author's results seem in many ways to justify his assumption of an unvarying solar curve in this region, it is known that in the far ultra violet (among other regions) the temporal variations in the solar curve are of enormous magnitude. Therefore, the final interpretation of these measurements of ozone, made indirectly by phototubes which solely determine energy reception in the near ultra violet, must await radiation measurements made by rockets or other more direct upper atmosphere probes.

Nevertheless, the definite positive correlation of the 8-km atmospheric temperature and negative correlation of the 16-km temperature with author's computed amount of ozone are extremely provocative. Their possible interpretation in terms of air currents at these high levels may find checks in other meteorological work.
Joanne Starr Malkus, USA

Lubrication; Bearings; Wear

2834. **F. Morel and G. Leroy, Tests on lubrication of friction shoes** (in French), C. R. Acad. Sci. 230, 174–175 (Jan. 1950).

Friction between a pivoted shoe and moving flat surface for a number of mineral oils used for pivots is measured as a function of speed and temperature. From 15 C to 30 C friction decreases as the speed increases in the low speed range, thus leading to relaxation oscillations. From 30 C to 60 C friction increases with speed to a maximum and then decreases slightly, but is much higher in value than at lower temperatures. At low temperatures, friction seems to correlate best with viscosity for a series of oils, while at higher temperatures it seems to depend more on oiliness, correlating with the saponification index.
J. T. Burwell, Jr., USA

2835. Gunther Cohn and Jess W. Oren, Film-pressure distribution in grease-lubricated journal bearings, Trans. Amer. Soc. mech. Engrs. 71, 555–560 (July 1949).

Under conditions of copious feed of a cup grease, measurements showed a circumferential and axial distribution of pressure in the load-carrying film of a grease-lubricated journal bearing which was qualitatively similar, although less peaked, than corresponding pressure distribution of an oil used in the bearing at same load and speed. The integrated grease pressure was found to closely equal the applied load. It was therefore concluded that in these experiments the grease operated hydrodynamically to support applied load. Speed affected pressure distribution by shifting both leading and trailing edges of grease film, but had little other effect.

John T. Burwell, Jr., USA

2836. Robert Schnurmann, Mechanism of friction in the "extreme-pressure" region, Proc. seventh int. Congr. appl. Mech. 4, 248–256 (1948).

Marine Engineering Problems

(See also Rev. 2829)

2837. J. Bleuzen, Study of turning tests at the Paris model basin, David Taylor Model Basin Transl. 222, 18 pp. (Sept. 1949).

Design and operation of the Paris turning basin for model ship-tests is summarized, with a few technical details. Tests of self-powered free-turning models and models towed by a rotating arm (diameter 65 meters; up to 3 rpm) are described very briefly. Projected instrumentation and future tests are surveyed.

A. O. Williams, Jr., USA

2838. C. J. G. Jensen, Ship structural members—part IV, Trans. N. E. Coast Instn. Engrs. Shipb. 65, no. 2, 99–116 (1948–1949).

Biomechanics

2839. B. Bresler and J. P. Frankel, The forces and moments in the leg during level walking, Trans. Amer. Soc. mech. Engrs. 72, 27–36 (Jan. 1950).

Mechanism of normal level walking is presented in terms of displacements of, and force systems at the leg joints. Data on four normal subjects were obtained from simultaneous recording of positions of leg in space and floor reactions during level walking. Mass moments of inertia of lower extremity were determined experimentally, and effects of gravity and inertia were included in the analysis. Forces and moments are presented in terms of space components referred to a system of horizontal and vertical orthogonal axes.

From author's summary

2840. Douglas G. Ellson and David Gibard, The application of operational analysis to human motor behavior, CADO tech. Data Dig. 14, 13–20 (Sept. 1, 1949).

Paper describes in elementary terms certain psychomotor problems and techniques of operational analysis of linear systems, and then proceeds to suggest the use of operational analysis in the human engineering field. Author's criterion of applicability is the linearity of the human as a servo. One reference to establish linearity is quoted: "The data indicate an approximately linear increase in rate of correction with increase in magnitude of displacement," which author takes to be evidence of linearity. A reference to other preliminary data is also cited to support this point. Author points out that "if *a* human being acts as a linear system, it is quite possible that he may be a *different* linear system in different situations"; "the human transmission system, even though it may be linear, has different characteristics as a function of practice"; and "operational analysis will also be concerned with individual differences."

Reviewer is inclined to sympathize with suggestion that the methods of operational calculus and servo analysis be applied to study of the human machine; but also to place more credence in experiments of Dr. F. V. Taylor, who has found that: (1) The subject never makes two responses exactly alike, and may even respond in the wrong direction and subsequently respond to that error; (2) one subject may respond with 2 to 4 times the accelerative response of another; (3) there is a nonlinear "range" effect in which response to a given step is influenced by relation of this error to other sizes in the series; (4) man has a time delay; (5) the correction motion is *not* controlled continuously by a feedback of one time delay, but intermittently; and (6) the length of open loop periods is, at least in part, a function of the rate of change of error.

Dr. Taylor's view [*Certain Characteristics of the Human Servo*, Psychology Section, Radio Division III, Naval Research Laboratory]: "this is not to state categorically that the methods of study-state analysis are utterly inappropriate . . . . when applied to man. It is to say, rather, that they must not be employed blindly. . . . . . It may be that further work will show that man's nonlinearities interact in a compensatory rather than an additive fashion. . . . . But even if this is not found, there is no need for overwhelming despair. Though man is complex, he is not chaotic. . . . It is likely that new measuring techniques will be required, and it is possible that new mathematical methods will have to be elaborated"—seems an eminently intelligent statement, and Ellson's suggestion, with these reservations, an eminently worthy one.

Robert R. Williamson, USA

Please Note

The Subject and Author Indexes to Applied Mechanics Reviews, Vol. 3, January–December, 1950, are to be published in separate pamphlet form and will be mailed together with the January, 1951, issue of Applied Mechanics Reviews.